## Contents

## PART I THREE CLASSES OF PROBLEMS IN MATHEMATICS, PHYSICS, AND ENGINEERING

Chapter 1. Simplest ill-posed problems	2
1.1. Statement of the problem. Examples	2
1.2. Definitions	6
1.3. Examples and approaches to solving ill-posed problems	7
1.4. Ill-posed problems of synthesis for optimum control systems	13
1.5. Ill-posed problems on finding eigenvalues for systems of linear	
homogeneous equations	22
1.6. Solution of systems of differential equations. Do solutions always	
depend on parameters continuously?	25
1.7. Conclusions	33
Chapter 2. Problems intermediate between well-	
and ill-posed problems	35
2.1. The third class of problems in mathematics, physics and engineer-	
ing, and its significance	35
2.2. Transformations equivalent in the classical sense	36
2.3. Discovered paradoxes	39
2.4. Transformations equivalent in the widened sense	40
2.5. Problems intermediate between well- and ill-posed problems	43
2.6. Applications to control systems and some other objects described	
by differential equations	48
2.7. Applications to practical computations	56
2.8. Conclusions from Chapters 1 and 2	62

vi Yu. P. Petrov and V. S. Sizikov. Well-posed, ill-posed, and	
Chapter 3. Change of sensitivity to measurement errors under integral transformations used in modeling of ships and marine control systems	64
3.1. Application of integral transformations to practical problems	64
3.2. Properties of correlation functions	68
3.3. Properties of spectra	73
3.4. Correctness of integral transformations	77
	80
3.5. Problems low sensitive to errors in the spectrum	86
3.6. Differentiation of distorted functions	1.002(102)
3.7. Prognostication	92
Bibliography to Part I	102
PART II	
Stable methods for solving inverse problems	
Chapter 4. Regular methods for solving ill-posed problems	107
4.1. Elements of functional analysis	107
4.2. Some facts from linear algebra	114
4.3. Basic types of equations and transformations	121
4.4. Well- and ill-posedness according to Hadamard	131
4.5. Classical methods for solving Fredholm integral equations of the	
first kind	
first kind	
	143
4.6. Gauss least-squares method and Moore-Penrose inverse-matrix	143 148
4.6. Gauss least-squares method and Moore-Penrose inverse-matrix method	143 148 155
4.6. Gauss least-squares method and Moore-Penrose inverse-matrix method	143 148 155
4.6. Gauss least-squares method and Moore-Penrose inverse-matrix method	143 148 155
4.6. Gauss least-squares method and Moore-Penrose inverse-matrix method	143 148 155 176
4.6. Gauss least-squares method and Moore-Penrose inverse-matrix method	143 148 155 176 187

225