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Lawrence Perko

Differential Equations and Dynamical Systems

With 177 Illustrations

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To my wife, Kathy, and children, Mary, Mike, Vince, Jenny and John, for all the joy they bring to my life.

Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM)* .

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Preface

This book covers those topics necessary for a clear understanding of the qualitative theory of ordinary differential equations. It is written for upper-division or first-year graduate students. It begins with a study of linear systems of ordinary differential equations, a topic already familiar to the student who has completed a first course in differential equations. An efficient method for solving any linear system of ordinary differential equations is presented in Chapter 1.

The major part of this book is devoted to a study of nonlinear systems of ordinary differential equations. Since most nonlinear differential equations cannot be solved, this book focuses on the qualitative or geometrical theory of nonlinear systems of differential equations originated by Henri Poincaré in his work on differential equations at the end of the nineteenth century. Our primary goal is to describe the qualitative behavior of the solution set of a given system of differential equations. In order to achieve this goal, it is first necessary to develop the local theory for nonlinear systems. This is done in Chapter 2 which includes the fundamental local existence–uniqueness theorem, the Hartman–Grobman Theorem and the Stable Manifold Theorem. These latter two theorems establish that the qualitative behavior of the solution set of a nonlinear system of ordinary differential equations near an equilibrium point is typically the same as the qualitative behavior of the solution set of the corresponding linearized system near the equilibrium point.

After developing the local theory, we turn to the global theory in Chapter 3. This includes a study of limit sets of trajectories and the behavior of trajectories at infinity. Some unsolved problems of current research interest are also presented in Chapter 3. For example, the Poincaré–Bendixson Theorem, established in Chapter 3, describes the limit sets of trajectories of two-dimensional systems; however, the limit sets of trajectories of three-dimensional (and higher dimensional) systems can be much more complicated and establishing the nature of these limit sets is a topic of current research interest in mathematics. In particular, higher dimensional systems can exhibit strange attractors and chaotic dynamics. All of the preliminary material necessary for studying these more advanced topics is contained in this textbook. This book can therefore serve as a springboard for those students interested in continuing their study of ordinary differential equations and dynamical systems. Chapter 3 ends with a technique for constructing the global phase portrait of a two-dimensional dynamical system. The global phase portrait describes the qualitative behavior of the solution set for all time. In general, this is as close as we can come to "solving" nonlinear systems.

In Chapter 4, we study systems of differential equations depending on a parameter. The question of particular interest is: For what values of the parameter does the global phase portrait of a dynamical system change its qualitative structure? The answer to this question forms the subject matter of bifurcation theory. An introduction to bifurcation theory is presented in Chapter 4 where we discuss bifurcations at nonhyperbolic equilibrium points and periodic orbits as well as Hopf bifurcations. Chapter 4 ends with a discussion of homoclinic loop bifurcations for planar systems and an introduction to tangential homoclinic bifurcations and the resulting chaotic dynamics that can occur in higher dimensional systems.

The prerequisites for studying differential equations and dynamical systems using this book are courses in linear algebra and real analysis. For example, the student should know how to find the eigenvalues and eigenvectors of a linear transformation represented by a square matrix and should be familiar with the notion of uniform convergence and related concepts. In using this book, the author hopes that the student will develop an appreciation for just how useful the concepts of linear algebra, real analysis and geometry are in developing the theory of ordinary differential equations and dynamical systems.

I would like to express my sincere appreciation to my colleague Terrence Blows for his many helpful suggestions which led to a substantially improved final version of this book. I would also like to thank Louella Holter for her patience and precision in typing the original manuscript.

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