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Graduate Texts in Mathematics

continued after index

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Analysis Now

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For
Oluf and *Cecilie*,
innocents at home

Preface

Mathematical method, as it applies in the natural sciences in particular, consists of solving a given problem (represented by a number of observed or observable data) by neglecting so many of the details (these are afterward termed "irrelevant") that the remaining part fits into an axiomatically established model. Each model carries a theory, describing the implicit features of the model and its relations to other models. The role of the mathematician (in this oversimplified description of our culture) is to maintain and extend the knowledge about the models and to create new models on demand.

Mathematical analysis, developed in the 18th and 19th centuries to solve dynamical problems in physics, consists of a series of models centered around the real numbers and their functions. As examples, we mention continuous functions, differentiable functions (of various orders), analytic functions, and integrable functions; all classes of functions defined on various subsets of euclidean space $\mathbb{R}^n$, and several classes also defined with vector values. Functional analysis was developed in the first third of the 20th century by the pioneering work of Banach, Hilbert, von Neumann, and Riesz, among others, to establish a model for the models of analysis. Concentrating on "external" properties of the classes of functions, these fit into a model that draws its axioms from (linear) algebra and topology. The creation of such "supermodels" is not a new phenomenon in mathematics, and, under the name of "generalization," it appears in every mathematical theory. But the users of the original models (astronomers, physicists, engineers, et cetera) naturally enough take a somewhat sceptical view of this development and complain that the mathematicians now are doing mathematics for its own sake. As a mathematician my reply must be that the abstraction process that goes into functional analysis is necessary to survey and to master the enormous material we have to handle. It is not obvious, for example, that a differential equation, a system of linear equations, and a problem in the calculus of variations have anything in common. A knowledge of operators on topological vector spaces gives, however, a basis of reference, within which the concepts of kernels, eigenvalues, and inverse transformations can be used on all three problems. Our critics, especially those well-meaning pedagogues, should come to realize that mathematics becomes simpler only through abstraction. The mathematics that represented the conceptual limit for the minds of Newton and Leibniz is taught regularly in our high schools, because we now have a clear (i.e. abstract) notion of a function and of the real numbers.

When this defense has been put forward for official use, we may admit in private that the wind is cold on the peaks of abstraction. The fact that the objects and examples in functional analysis are themselves mathematical theories makes communication with nonmathematicians almost hopeless and deprives us of the feedback that makes mathematics more than an aesthetical play with axioms. (Not that this aspect should be completely neglected.) The dichotomy between the many small and directly applicable models and the large, abstract supermodel cannot be explained away. Each must find his own way between Scylla and Charybdis.

The material contained in this book falls under Kelley's label: What Every Young Analyst Should Know. That the young person should know more (e.g. more about topological vector spaces, distributions, and differential equations) does not invalidate the first commandment. The book is suitable for a two-semester course at the first year graduate level. If time permits only a one-semester course, then Chapters 1, 2, and 3 is a possible choice for its content, although if the level of ambition is higher, 4.1–4.4 may be substituted for 3.3–3.4. Whatever choice is made, there should be time for the student to do some of the exercises attached to every section in the first four chapters. The exercises vary in the extreme from routine calculations to small guided research projects. The two last chapters may be regarded as huge appendices, but with entirely different purposes. Chapter 5 on (the spectral theory of) unbounded operators builds heavily upon the material contained in the previous chapters and is an end in itself. Chapter 6 on integration theory depends only on a few key results in the first three chapters (and may be studied simultaneously with Chapters 2 and 3), but many of its results are used implicitly (in Chapters 2–5) and explicitly (in Sections 4.5–4.7 and 5.3) throughout the text.

This book grew out of a course on the Fundamentals of Functional Analysis given at The University of Copenhagen in the fall of 1982 and again in 1983. The primary aim is to give a concentrated survey of the tools of modern analysis. Within each section there are only a few main results—labeled theorems—and the remaining part of the material consists of supporting lemmas, explanatory remarks, or propositions of secondary importance. The style of writing is of necessity compact, and the reader must be prepared to supply minor details in some arguments. In principle, though, the book is "self-contained." However, for convenience, a list of classic or established textbooks, covering (parts of) the same material, has been added. In the Bibliography the reader will also find a number of original papers, so that she can judge for herself "wie es eigentlich gewesen."

Several of my colleagues and students have read (parts of) the manuscript and offered valuable criticism. Special thanks are due to B. Fuglede, G. Grubb, E. Kehlet, K.B. Laursen, and F. Topsøe.

The title of the book may convey the feeling that the message is urgent and the medium indispensable. It may as well be construed as an abbreviation of the scholarly accurate heading: Analysis based on Norms, Operators, and Weak topologies.

Copenhagen — Gert Kjærgård Pedersen

Preface to the Second Printing

Harald Bohr is credited with saying that if mathematics does not teach us to think correctly, at least it teaches us how easy it is to think incorrectly. Certainly an embarrassing number of mistakes and misprints in this book have been brought to my attention during the past five years. Also, more or less desperate students have pointed out many phrases and formulations that made little sense without further explanation. I am deeply grateful to Springer-Verlag for allowing the numerous corrections in this revised second printing, and hope that it will be of improved service to the fastidious mathematicians it was aimed for.

GKP

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