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A. Pazy

Semigroups of Linear Operators and Applications to Partial Differential Equations

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Preface to the Second Printing

This second printing of the book contains a few minor changes and corrections. It is a pleasure for me to thank Peter Hess, Gunter Lumer, R. de Roo, and Hans Sager for drawing my attention to many misprints and some errors.

I am especially indebted to Shinnosuke Oharu, who went through the whole book and recommended many valuable clarifications, modifications, and corrections.

A. PAZY

Preface to the First Printing

The aim of this book is to give a simple and self-contained presentation of the theory of semigroups of bounded linear operators and its applications to partial differential equations.

The book is a corrected and expanded version of a set of lecture notes which I wrote at the University of Maryland in 1972–1973. The first three chapters present a short account of the abstract theory of semigroups of bounded linear operators. Chapters 4 and 5 give a somewhat more detailed study of the abstract Cauchy problem for autonomous and nonautonomous linear initial value problems, while Chapter 6 is devoted to some abstract nonlinear initial value problems. The first six chapters are self-contained and the only prerequisite needed is some elementary knowledge of functional analysis. Chapters 7 and 8 present applications of the abstract theory to concrete initial value problems for linear and nonlinear partial differential equations. Some of the auxiliary results from the theory of partial differential equations used in these chapters are stated without proof. References where the proofs can be found are given in the bibliographical notes to these chapters.

I am indebted to many good friends who read the lecture notes on which this book is based, corrected errors, and suggested improvements. In particular I would like to express my thanks to H. Brezis, M.G. Crandall, and P. Rabinowitz for their valuable advice, and to Danit Sharon for the tedious work of typing the manuscript.

A. PAZY

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