

Michael E. Taylor

Partial Differential Equations II

Qualitative Studies of Linear Equations

With 13 Illustrations



Springer

Michael E. Taylor
Department of Mathematics
University of North Carolina
Chapel Hill, NC 27599
USA

Editors

Jerrold E. Marsden
Control and Dynamical Systems, 104-44
California Institute of Technology
Pasadena, CA 91125
USA

L. Sirovich
Division of Applied Mathematics
Brown University
Providence, RI 02912
USA

Mathematics Subject Classification (1991): 35-01, 35S05, 35P05, 35P25, 58G32, 35N15

Library of Congress Cataloging-in-Publication Data

Taylor, Michael Eugene, 1946–

Partial differential equations / Michael E. Taylor.

p. cm. — (Applied mathematical sciences ; v. 115–117)

Includes bibliographical references and index.

Contents: 1. Basic theory — 2. Qualitative studies of linear equations — 3. Nonlinear equations.

1. Differential equations, Partial. I. Title. II. Series:

Applied mathematical sciences (Springer-Verlag New York Inc.) ; v. 115–117.

QA1.A647 vol. 115–117

[QA374]

510 s—dc20

[515'.353]

95-54104

Printed on acid-free paper.

© 1996 Springer Science+Business Media New York

Originally published by Springer-Verlag New York, Inc. in 1996.

Softcover reprint of the hardcover 1st edition 1996

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher Springer Science+Business Media, LLC, except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production coordinated by Chernow Editorial Services, Inc. and managed by Bill Imbornoni; manufacturing supervised by Jeffrey Taub.

Camera-ready copy prepared from the author's $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{T}\mathcal{E}\mathcal{X}$ files.

9 8 7 6 5 4 3 2 1

SPIN 10523995

ISBN 978-1-4757-4189-6

ISBN 978-1-4757-4187-2 (eBook)

DOI 10.1007/978-1-4757-4187-2

Contents

Contents of Volumes I and III	xi
--------------------------------------	-----------

Introduction	xiii
---------------------	-------------

7 Pseudodifferential Operators	1
---------------------------------------	----------

Introduction	1
1 The Fourier integral representation and symbol classes	2
2 Schwartz kernels of pseudodifferential operators	5
3 Adjoints and products	9
4 Elliptic operators and parametrices	14
5 L^2 -estimates	16
6 Gårding's inequality	19
7 Hyperbolic evolution equations	21
8 Egorov's theorem	24
9 Microlocal regularity	26
10 Operators on manifolds	30
11 The method of layer potentials	33
12 Parametrix for regular elliptic boundary problems	42
13 Parametrix for the heat equation	50
14 The Weyl calculus	60
References	70

8 Spectral Theory	74
--------------------------	-----------

Introduction	74
1 The spectral theorem	75
2 Self-adjoint differential operators	82
3 Heat asymptotics and eigenvalue asymptotics	88
4 The Laplace operator on S^n	94
5 The Laplace operator on hyperbolic space	103
6 The harmonic oscillator	105
7 The quantum Coulomb problem	113
8 The Laplace operator on cones	126
References	142

9	Scattering by Obstacles	145
	Introduction	145
1	The scattering problem	147
2	Eigenfunction expansions	155
3	The scattering operator	161
4	Connections with the wave equation	165
5	Wave operators	172
6	Translation representations and the Lax-Phillips semigroup	
	$Z(t)$	177
7	Integral equations and scattering poles	184
8	Trace formulas; the scattering phase	197
9	Scattering by a sphere	203
10	Inverse problems I	211
11	Inverse problems II	217
12	Scattering by rough obstacles	227
A	Lidskii's trace theorem	235
	References	237
10	Dirac Operators and Index Theory	241
	Introduction	241
1	Operators of Dirac type	242
2	Clifford algebras	248
3	Spinors	253
4	Weitzenböck formulas	258
5	Index of Dirac operators	264
6	Proof of the local index formula	267
7	The Chern-Gauss-Bonnet theorem	273
8	Spin^c manifolds	277
9	The Riemann-Roch theorem	281
10	Direct attack in 2-D	292
	References	298
11	Brownian Motion and Potential Theory	301
	Introduction	301
1	Brownian motion and Wiener measure	303
2	The Feynman-Kac formula	310
3	The Dirichlet problem and diffusion on domains with	
	boundary	314
4	Martingales, stopping times, and the strong Markov property	322
5	First exit time and the Poisson integral	331
6	Newtonian capacity	335
7	Stochastic integrals	349
8	Stochastic integrals, II	358

9	Stochastic differential equations	364
10	Application to equations of diffusion	371
A	The Trotter product formula	380
	References	386
12	The $\bar{\partial}$-Neumann Problem	389
	Introduction	389
A	Elliptic complexes	391
1	The $\bar{\partial}$ -complex	396
2	Morrey's inequality, the Levi form, and strong pseudoconvexity	399
3	The $\frac{1}{2}$ -estimate and some consequences	402
4	Higher-order subelliptic estimates	406
5	Regularity via elliptic regularization	409
6	The Hodge decomposition and the $\bar{\partial}$ -equation	412
7	The Bergman projection and Toeplitz operators	415
8	The $\bar{\partial}$ -Neumann problem on $(0, q)$ -forms	421
9	Reduction to pseudodifferential equations on the boundary	429
10	The $\bar{\partial}$ -equation on complex manifolds and almost complex manifolds	440
B	Complements on the Levi form	449
C	The Neumann operator for the Dirichlet problem	453
	References	456
C	Connections and Curvature	460
	Introduction	460
1	Covariant derivatives and curvature on general vector bundles	461
2	Second covariant derivatives and covariant-exterior derivatives	466
3	The curvature tensor of a Riemannian manifold	468
4	Geometry of submanifolds and subbundles	479
5	The Gauss-Bonnet theorem for surfaces	492
6	The principal bundle picture	502
7	The Chern-Weil construction	509
8	The Chern-Gauss-Bonnet theorem	514
	References	522
	Index	525