

Karl-Rudolf Koch

Parameter Estimation and Hypothesis Testing in Linear Models

Second, updated and enlarged Edition

With 20 Figures



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Contents

Introduction	1
1 Vector and Matrix Algebra	3
1.1 Sets and Fields	3
1.1.1 Notion of Sets	3
1.1.2 Composition of Sets	4
1.1.3 Relations	4
1.1.4 Field of Real Numbers	5
1.2 Vector Algebra	6
1.2.1 Definition of Vectors and Vector Space	6
1.2.2 Linear Dependence and Basis of a Vector Space	8
1.2.3 Inner Product and Euclidian Space	11
1.2.4 Orthogonal Subspaces	11
1.3 Matrices	13
1.3.1 Matrix Definition and Matrix Composition	13
1.3.2 Rank of a Matrix	19
1.3.3 Computation of Inverse Matrices	23
1.3.4 Matrix Identities	33
1.3.5 Column Space and Null Space of a Matrix	34
1.3.6 Determinants	36
1.3.7 Trace of a Matrix and Vector Representation	40
1.4 Quadratic Forms	41
1.4.1 Transformations	41
1.4.2 Eigenvalues and Eigenvectors	44
1.4.3 Definite Matrices	46
1.5 Generalized Inverses	48
1.5.1 Right Inverse and Left Inverse	48
1.5.2 Idempotent Matrices	48
1.5.3 Generalized Inverse, Reflexive Generalized Inverse and Pseudoinverse	50
1.5.4 Systems of Linear Equations	54
1.5.5 Generalized Inverses of Symmetrical Matrices	57
1.5.6 Properties of the Pseudoinverse and of a Special Symmetrical Reflexive Generalized Inverse	62
1.6 Projections	64
1.6.1 General Projections	64
1.6.2 Orthogonal Projections	65
1.7 Differentiation and Integration of Vectors and Matrices	66

1.7.1	Extrema of Functions	66
1.7.2	Derivatives of Special Functions	68
1.7.3	Integration and Transformation of Variables	71
2	Probability Theory	75
2.1	Probability	75
2.1.1	Introduction	75
2.1.2	Random Events	77
2.1.3	Axioms of Probability	78
2.1.4	Conditional Probability and Bayes' Formula	79
2.1.5	Independent Events	81
2.2	Random Variable	81
2.2.1	Definition	81
2.2.2	Distribution Function	82
2.2.3	Discrete and Continuous Random Variable	83
2.2.4	Binomial Distribution and Poisson Distribution	86
2.2.5	Multidimensional Continuous Random Variable	88
2.2.6	Marginal Distribution	90
2.2.7	Conditional Distribution	90
2.2.8	Independent Random Variables	92
2.2.9	Transformation of Variables	93
2.3	Expected Values and Moments of Random Variables	93
2.3.1	Expectation	93
2.3.2	Multivariate Moments	96
2.3.3	Covariance Matrix and Covariances of Random Vectors	99
2.3.4	Moment Generating Function	106
2.4	Univariate Distributions	107
2.4.1	Normal Distribution	107
2.4.2	Derivation of the Normal Distribution as Distribution of Observational Errors	111
2.4.3	Gamma Distribution	112
2.4.4	Derivation of the Gamma Distribution as Waiting-Time Distribution	114
2.4.5	Beta Distribution	115
2.5	Multivariate Normal Distribution	117
2.5.1	Definition and Derivation	117
2.5.2	Moment Generating Function for the Normal Distribution	118
2.5.3	Marginal Distribution and Conditional Distribution	120
2.5.4	Independence of Normally Distributed Random Variables	122
2.5.5	Linear Functions of Normally Distributed Random Variables	122
2.5.6	Sum of Normally Distributed Random Variables	123
2.6	Test Distributions for Univariate Models	124

2.6.1	χ^2 -Distribution	124
2.6.2	Non-Central χ^2 -Distribution	126
2.6.3	F -Distribution	128
2.6.4	Non-Central F -Distribution	130
2.6.5	t -Distribution	132
2.7	Quadratic Forms	133
2.7.1	Expected Value and Covariance	133
2.7.2	Distribution of the Quadratic Form	135
2.7.3	Independence of Two Quadratic Forms	136
2.7.4	Independence of a Linear and a Quadratic Form	136
2.8	Test Distributions for Multivariate Models	137
2.8.1	Wishart Distribution	137
2.8.2	Derivation of the Wishart Distribution	137
2.8.3	Distribution of the Sum of Wishart Matrices	140
2.8.4	Distribution of the Transformed Wishart Matrix	140
2.8.5	Distribution of Matrices of Quadratic Forms and Independence of Wishart Matrices	141
2.8.6	Distribution of the Ratio of the Determinants of Two Wishart Matrices	143
2.8.7	Distribution of Special Functions of Wishart Matrices	146
3	Parameter Estimation in Linear Models	149
3.1	Methods of Estimating Parameters	150
3.1.1	Point Estimation	150
3.1.2	Best Unbiased Estimation	150
3.1.3	Method of Least Squares	152
3.1.4	Maximum-Likelihood Method	153
3.2	Gauss-Markoff Model	153
3.2.1	Definition and Linearization	153
3.2.2	Best Linear Unbiased Estimation	156
3.2.3	Method of Least Squares	158
3.2.4	Maximum-Likelihood Method	161
3.2.5	Unbiased Estimation of the Variance of Unit Weight	162
3.2.6	Numerical Computation of the Estimates and Their Covariances	165
3.2.7	Gauss-Markoff Model with Constraints	170
3.2.8	Recursive Parameter Estimation	177
3.2.9	Deviations From the Model	178
3.3	Gauss-Markoff Model not of Full Rank	181
3.3.1	Method of Least Squares and Maximum-Likelihood Method	181
3.3.2	Estimable Functions	183
3.3.3	Projected Parameters as Estimable Functions	185
3.3.4	Gauss-Markoff Model not of Full Rank with Constraints	193

3.4	Special Gauss-Markoff Models	197
3.4.1	Polynomial Model	197
3.4.2	Analysis of Variance	200
3.4.3	Parameter Estimation for the Analysis of Variance by a Symmetrical Reflexive Generalized Inverse	204
3.4.4	Analysis of Covariance	207
3.4.5	Gauss-Markoff Model for Outliers in the Observations	208
3.5	Generalized Linear Models	210
3.5.1	Regression Model	210
3.5.2	Mixed Model	212
3.5.3	Best Linear Unbiased Estimation in the Mixed Model	214
3.5.4	Method of Least Squares and Maximum-Likelihood Method for the Mixed Model	216
3.5.5	Model of the Adjustment with Condition Equations	220
3.5.6	Prediction and Filtering	221
3.6	Estimation of Variance and Covariance Components	225
3.6.1	Best Invariant Quadratic Unbiased Estimation	225
3.6.2	Locally Best Estimation	229
3.6.3	Iterated Estimates	233
3.6.4	Best Unbiased Estimation of the Variance of Unit Weight	237
3.7	Multivariate Parameter Estimation	238
3.7.1	Multivariate Gauss-Markoff Model	238
3.7.2	Estimation of the Vectors of Parameters	240
3.7.3	Estimation of the Covariance Matrix	241
3.7.4	Numerical Computation of the Estimates and Incomplete Multivariate Models	246
3.7.5	Special Model for Estimating Covariance Matrices and Estimation of Covariances for Stochastic Processes	250
3.7.6	Multivariate Model with Constraints	253
3.8	Robust Parameter Estimation	255
3.8.1	Choice of the Score Function	255
3.8.2	Robust M-Estimation	255
3.8.3	M-Estimation of Huber	258
3.8.4	L_p -Norm Estimation	261
3.8.5	Leverage Points	263
3.8.6	Modified M-Estimation of Huber	265
3.8.7	Method of Rousseeuw	268
4	Hypothesis Testing, Interval Estimation and Test for Outliers	271
4.1	Distributions Based on Normally Distributed Observations	272
4.1.1	Distributions of Functions of the Residuals in the Univariate Model	272

4.1.2	Distributions of the Covariance Matrices Estimated in the Multivariate Model	274
4.2	Test of Hypotheses	276
4.2.1	Methods for Hypothesis Testing and Likelihood Ratio Test	276
4.2.2	Test of a General Hypothesis in the Univariate Gauss-Markoff Model	279
4.2.3	Special Hypotheses	283
4.2.4	Hypothesis Testing for the Variance of Unit Weight	286
4.2.5	Test of a General Hypothesis in the Multivariate Gauss-Markoff Model	288
4.2.6	Hypothesis of the Identity of a Covariance Matrix with a Given Matrix	294
4.3	Interval Estimation	296
4.3.1	Confidence Intervals	296
4.3.2	Confidence Intervals for Parameters, for Linear Functions of the Parameters and Confidence Hyperellipsoids	296
4.3.3	Confidence Interval for the Variance of Unit Weight	301
4.4	Test for Outliers	302
4.4.1	Hypothesis Testing for Outliers	302
4.4.2	Test for One Outlier	304
4.4.3	Percentage Points of the Test for Outliers	306
4.4.4	Measures of Reliability	307
References		311
Index		327