

Sources
in the History of Mathematics and
Physical Sciences

8

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Pappus of Alexandria

Book 7 of the
Collection

Part 1. Introduction, Text, and Translation

Edited
With Translation and Commentary by
Alexander Jones

In Two Parts
With 308 Figures



Springer Science+Business Media, LLC

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AMS Classification: 01A20

Library of Congress Cataloging in Publication Data

Pappus, of Alexandria.

Book 7 of the Collection.

(Sources in the history of mathematics and physical sciences ; 8)
English and Greek.

Revision of thesis (Ph. D.)—Brown University, 1985.

Bibliography: p.

Includes indexes.

Contents: pt. 1. Introduction, text, and translation—
pt. 2. Commentary, index, and figures.

1. Mathematics, Greek. I. Jones, Alexander.

II. Title. III. Title: Book seven of the Collection.

IV. Series.

QA22.P3713 1986 516.2 85-27788

© 1986 by Springer Science+Business Media New York

Originally published by Springer-Verlag New York Inc. in 1986

Softcover reprint of the hardcover 1st edition 1986

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9 8 7 6 5 4 3 2 1

ISBN 978-1-4612-9355-2 ISBN 978-1-4612-4908-5 (eBook)

DOI 10.1007/978-1-4612-4908-5

TO MY PARENTS

Preface

The seventh book of Pappus's *Collection*, his commentary on the Domain (or Treasury) of Analysis, figures prominently in the history of both ancient and modern mathematics: as our chief source of information concerning several lost works of the Greek geometers Euclid and Apollonius, and as a book that inspired later mathematicians, among them Viète, Newton, and Chasles, to original discoveries in their pursuit of the lost science of antiquity. This presentation of it is concerned solely with recovering what can be learned from Pappus about Greek mathematics. The main part of it comprises a new edition of Book 7; a literal translation; and a commentary on textual, historical, and mathematical aspects of the book. It proved to be convenient to divide the commentary into two parts, the notes to the text and translation, and essays about the lost works that Pappus discusses.

The first function of an edition of this kind is, not to expose new discoveries, but to present a reliable text and organize the accumulated knowledge about it for the reader's convenience. Nevertheless there are novelties here. The text is based on a fresh transcription of Vat. gr. 218, the archetype of all extant manuscripts, and in it I have adopted numerous readings, on manuscript authority or by emendation, that differ from those of the old edition of Hultsch. Moreover, many difficult parts of the work have received little or no commentary hitherto. In particular I believe that more sense can be recovered from several problematic passages in the important first part of the book than has been recognized. The account of the evolution and vicissitudes of the text, from its composition to the Renaissance, is largely new. In treating the lost works of Apollonius and Euclid, where so much has been done between the times of Maurolico and Zeuthen, my main work was to select what seemed to be valid scholarship; the remainder, if mentioned at all, had to be ruthlessly relegated to footnotes, without regard for intrinsic merit.

This edition is a revision of my doctoral dissertation in the Department of History of Mathematics at Brown University, which was submitted in April 1985. It was stored on and printed by Brown University's computer facilities, using experimental laser-printer typesetting software. Some minor typographical infelicities, for example the lack of an iota subscript, are I hope outweighed by the reduced cost of production. I am entirely responsible for typographical and other errors.

I have to thank the Biblioteca Apostolica Vaticana for access to its facilities and collections, and providing, through my teacher Gerald Toomer, a microfilm of the archetype. I have also profited from research in the Biblioteca Ambrosiana, Milan; the Newberry Library, Chicago; the libraries of the University of British Columbia and Simon Fraser University; and above all the libraries of Brown University. During the writing of the

dissertation I held a doctoral fellowship from the Social Sciences and Humanities Research Council of Canada. The History of Mathematics Department provided a truly congenial home for four years; I mention with special gratitude the often manifested hospitality of the late Professor A. J. Sachs and Mrs J. Sachs, and many kindnesses of Professor O. Neugebauer. A summer stipend from the History of Mathematics Department enabled me to spend two months during the Summer of 1984 in Italy palpating the past. For various suggestions, information, and corrections I am indebted to Professors J. L. Berggren, A. L. Boegehold, David Pingree (who also proof-read the Greek text expertly), D. T. Whiteside, and Mr N. G. Wilson. Dr Jan Hogendijk, surpassing his function as reader of the dissertation, rescued me from numerous mathematical and logical morasses. Many of my notes on Pappus's mathematics are the better for his suggestions, and the essays (especially those on the *Porisms* and the loci) were enormously improved, in form and content, under his guidance. He also generously allowed me to read the results of his researches into the traces of lost works of Apollonius in Arabic sources; since these are, at the time of writing, not published, I have limited myself to mentioning the existence of relevant fragments at appropriate points in the essay on Apollonius. My debt to Gerald Toomer extends throughout the book, every page of which (in its earlier version) he read with the greatest care. He suggested the edition in the first place, and I can only hope that a little of his learning is reflected in it.

Providence,
September 1985.

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