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Level Set Methods and Dynamic Implicit Surfaces

With 99 Figures, Including 24 in Full Color



Springer

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Contents

Preface	vii
Color Insert	(facing page 146)
I Implicit Surfaces	1
1 Implicit Functions	3
1.1 Points	3
1.2 Curves	4
1.3 Surfaces	7
1.4 Geometry Toolbox	8
1.5 Calculus Toolbox	13
2 Signed Distance Functions	17
2.1 Introduction	17
2.2 Distance Functions	17
2.3 Signed Distance Functions	18
2.4 Examples	19
2.5 Geometry and Calculus Toolboxes	21
II Level Set Methods	23
3 Motion in an Externally Generated Velocity Field	25
3.1 Convection	25

3.2	Upwind Differencing	29
3.3	Hamilton-Jacobi ENO	31
3.4	Hamilton-Jacobi WENO	33
3.5	TVD Runge-Kutta	37
4	Motion Involving Mean Curvature	41
4.1	Equation of Motion	41
4.2	Numerical Discretization	44
4.3	Convection-Diffusion Equations	45
5	Hamilton-Jacobi Equations	47
5.1	Introduction	47
5.2	Connection with Conservation Laws	48
5.3	Numerical Discretization	49
5.3.1	Lax-Friedrichs Schemes	50
5.3.2	The Roe-Fix Scheme	52
5.3.3	Godunov's Scheme	54
6	Motion in the Normal Direction	55
6.1	The Basic Equation	55
6.2	Numerical Discretization	57
6.3	Adding a Curvature-Dependent Term	59
6.4	Adding an External Velocity Field	59
7	Constructing Signed Distance Functions	63
7.1	Introduction	63
7.2	Reinitialization	64
7.3	Crossing Times	65
7.4	The Reinitialization Equation	65
7.5	The Fast Marching Method	69
8	Extrapolation in the Normal Direction	75
8.1	One-Way Extrapolation	75
8.2	Two-Way Extrapolation	76
8.3	Fast Marching Method	76
9	Particle Level Set Method	79
9.1	Eulerian Versus Lagrangian Representations	79
9.2	Using Particles to Preserve Characteristics	82
10	Codimension-Two Objects	87
10.1	Intersecting Two Level Set Functions	87
10.2	Modeling Curves in \mathbb{R}^3	87
10.3	Open Curves and Surfaces	90
10.4	Geometric Optics in a Phase-Space-Based Level Set Framework	90

III	Image Processing and Computer Vision	95
11	Image Restoration	97
11.1	Introduction to PDE-Based Image Restoration	97
11.2	Total Variation-Based Image Restoration	99
11.3	Numerical Implementation of TV Restoration	103
12	Snakes, Active Contours, and Segmentation	119
12.1	Introduction and Classical Active Contours	119
12.2	Active Contours Without Edges	121
12.3	Results	124
12.4	Extensions	124
13	Reconstruction of Surfaces from Unorganized Data Points	139
13.1	Introduction	139
13.2	The Basic Model	140
13.3	The Convection Model	142
13.4	Numerical Implementation	142
IV	Computational Physics	147
14	Hyperbolic Conservation Laws and Compressible Flow	149
14.1	Hyperbolic Conservation Laws	149
14.1.1	Bulk Convection and Waves	150
14.1.2	Contact Discontinuities	151
14.1.3	Shock Waves	152
14.1.4	Rarefaction Waves	153
14.2	Discrete Conservation Form	154
14.3	ENO for Conservation Laws	155
14.3.1	Motivation	155
14.3.2	Constructing the Numerical Flux Function	157
14.3.3	ENO-Roe Discretization (Third-Order Accurate)	158
14.3.4	ENO-LLF Discretization (and the Entropy Fix)	159
14.4	Multiple Spatial Dimensions	160
14.5	Systems of Conservation Laws	160
14.5.1	The Eigensystem	161
14.5.2	Discretization	162
14.6	Compressible Flow Equations	163
14.6.1	Ideal Gas Equation of State	164
14.6.2	Eigensystem	164
14.6.3	Numerical Approach	165

15 Two-Phase Compressible Flow	167
15.1 Introduction	167
15.2 Errors at Discontinuities	168
15.3 Rankine-Hugoniot Jump Conditions	169
15.4 Nonconservative Numerical Methods	171
15.5 Capturing Conservation	172
15.6 A Degree of Freedom	172
15.7 Isobaric Fix	173
15.8 Ghost Fluid Method	175
15.9 A Robust Alternative Interpolation	183
16 Shocks, Detonations, and Deflagrations	189
16.1 Introduction	189
16.2 Computing the Velocity of the Discontinuity	190
16.3 Limitations of the Level Set Representation	191
16.4 Shock Waves	191
16.5 Detonation Waves	193
16.6 Deflagration Waves	195
16.7 Multiple Spatial Dimensions	196
17 Solid-Fluid Coupling	201
17.1 Introduction	201
17.2 Lagrange Equations	203
17.3 Treating the Interface	204
18 Incompressible Flow	209
18.1 Equations	209
18.2 MAC Grid	210
18.3 Projection Method	212
18.4 Poisson Equation	213
18.5 Simulating Smoke for Computer Graphics	214
19 Free Surfaces	217
19.1 Description of the Model	217
19.2 Simulating Water for Computer Graphics	218
20 Liquid-Gas Interactions	223
20.1 Modeling	223
20.2 Treating the Interface	224
21 Two-Phase Incompressible Flow	227
21.1 Introduction	227
21.2 Jump Conditions	230
21.3 Viscous Terms	232
21.4 Poisson Equation	235

22 Low-Speed Flames	239
22.1 Reacting Interfaces	239
22.2 Governing Equations	240
22.3 Treating the Jump Conditions	241
23 Heat Flow	249
23.1 Heat Equation	249
23.2 Irregular Domains	250
23.3 Poisson Equation	251
23.4 Stefan Problems	254
References	259
Index	271