Applied Mathematical Sciences

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Singular Perturbation Methods for Ordinary Differential Equations

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Preface

This book results from various lectures given in recent years. Early drafts were used for several single semester courses on singular perturbation methods given at Rensselaer, and a more complete version was used for a one year course at the Technische Universität Wien. Some portions have been used for short lecture series at Universidad Central de Venezuela, West Virginia University, the University of Southern California, the University of California at Davis, East China Normal University, the University of Texas at Arlington, Universitá di Padova, and the University of New Hampshire, among other places. As a result, I've obtained lots of valuable feedback from students and listeners, for which I am grateful. This writing continues a pattern. Earlier lectures at Bell Laboratories, at the University of Edinburgh and New York University, and at the Australian National University led to my earlier works (1968, 1974, and 1978). All seem to have been useful for the study of singular perturbations, and I hope the same will be true of this monograph. I've personally learned much from reading and analyzing the works of others, so I would especially encourage readers to treat this book as an introduction to a diverse and exciting literature.

The topic coverage selected is personal and reflects my current opinions. An attempt has been made to encourage a consistent method of approaching problems, largely through correcting outer limits in regions of rapid change. Formal proofs of correctness are not emphasized. Instead, some nontrivial applications are included. In a first reading, one might be advised to skip to a later section from time to time (depending on one's background and interests). Little effort has been made to provide an exhaustive list of references. There are simply too many relevant works, so I've merely tried to be representative, emphasizing textbooks to some extent. Despite the special perspective, I hope the book will prove useful in teaching readers how to solve applied problems in a variety of contexts.

Many individuals deserve thanks for helping me understand many aspects of asymptotic analysis and its applications. My presentation here has been influenced by their papers and sometimes by their answers to specific questions. This has been especially true recently concerning historical information used to prepare the Appendix. Others helped generously

through encouragement, hospitality, and support. In particular, much of the research reported was supported by the National Science Foundation, the U. S. Army Research Office, the Air Force Office of Scientific Research, and the Technical University of Vienna. Peggy Lashway and Jacques LaForgue worked especially hard to help me prepare the manuscript.

To all, many thanks.

Robert E. O'Malley, Jr.
Seattle, Winter 1990

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