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**Nonlinear Oscillations,
Dynamical Systems,
and Bifurcations
of Vector Fields**

With 206 Illustrations



Springer

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