

ENCYCLOPEDIA OF MATHEMATICS AND ITS
APPLICATIONS

Absolute Measurable Spaces

TOGO NISHIURA



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press

The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521875561

© T. Nishiura 2008

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2008

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

ISBN 978-0-521-87556-1 hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

<i>Preface</i>	<i>page</i> ix
<i>Acknowledgements</i>	xiii
1 The absolute property	1
1.1 Absolute measurable spaces	1
1.2 Absolute null spaces	7
1.3 Existence of absolute null spaces	10
1.4 Grzegorek's cardinal number κ_G	18
1.5 More on existence of absolute null spaces	24
1.6 Comments	26
Exercises	28
2 The universally measurable property	30
2.1 Universally measurable sets	31
2.2 Positive measures	35
2.3 Universally measurable maps	37
2.4 Symmetric difference of Borel and null sets	39
2.5 Early results	42
2.6 The homeomorphism group of $[0, 1]$	43
2.7 The group of \mathfrak{B} -homeomorphisms	46
2.8 Comments	49
Exercises	52
3 The homeomorphism group of X	53
3.1 A metric for $\text{HOMEO}(X)$	54
3.2 General properties	56
3.3 One-dimensional spaces	57
3.4 The Oxtoby–Ulam theorem	61
3.5 n -dimensional manifolds	73
3.6 The Hilbert cube	76
3.7 Zero-dimensional spaces	82

3.8	Other examples	88
3.9	Comments	90
	Exercises	97
4	Real-valued functions	99
4.1	A solution to Goldman's problem	100
4.2	Differentiability and \mathfrak{B} -maps	103
4.3	Radon–Nikodym derivative and Oxtoby–Ulam theorem	105
4.4	Zahorski spaces	112
4.5	Bruckner–Davies–Goffman theorem	115
4.6	Change of variable	126
4.7	Images of Lusin sets	128
4.8	Comments	130
	Exercises	134
5	Hausdorff measure and dimension	136
5.1	Universally null sets in metric spaces	136
5.2	A summary of Hausdorff dimension theory	137
5.3	Cantor cubes	139
5.4	Zindulka's theorem	143
5.5	Analytic sets in \mathbb{R}^n	146
5.6	Zindulka's opaque sets	151
5.7	Comments	154
	Exercises	156
6	Martin axiom	157
6.1	CH and universally null sets: a historical tour	157
6.2	Absolute null space and cardinal numbers	165
6.3	Consequences of the Martin axiom	168
6.4	Topological dimension and MA	171
6.5	Comments	173
	Exercises	178
Appendix A	Preliminary material	179
A.1	Complete metric spaces	179
A.2	Borel measurable maps	182
A.3	Totally imperfect spaces	185
A.4	Complete Borel measure spaces	186
A.5	The sum of Borel measures	192
A.6	Zahorski spaces	193
A.7	Purves's theorem	194
A.8	Comments	203
	Exercises	203

Appendix B Probability theoretic approach	204
B.1 Basic definitions	204
B.2 Separable metrizable	206
B.3 Shortt's observation	208
B.4 Lusin measurable space	210
B.5 Comments	212
Exercises	213
Appendix C Cantor spaces	214
C.1 Closed and open sets	215
C.2 A metric for $k^{\mathbb{N}}$	217
C.3 Bernoulli measures	219
C.4 Uniform Bernoulli distribution	220
C.5 Binomial Bernoulli distribution	221
C.6 Linear ordering of $\{0, 1\}^{\mathbb{N}}$ and good measures	230
C.7 Refinable numbers	233
C.8 Refinable numbers and good measures	239
C.9 Comments	240
Exercises	242
Appendix D Dimensions and measures	244
D.1 Topological dimension	244
D.2 Measure theoretical dimension	246
D.3 Zindulka's dimension theorem	249
D.4 Geometric measure theory	253
D.5 Marstrand's theorem	255
Exercises	257
<i>Bibliography</i>	258
<i>Notation index</i>	267
<i>Author index</i>	270
<i>Subject index</i>	272