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Analytic Number Theory

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Introduction and Dedication

This book is dedicated to Paul Erdős, the greatest mathematician I have ever known, whom it has been my rare privilege to consider colleague, collaborator, and dear friend.

I like to think that Erdős, whose mathematics embodied the principles which have impressed themselves upon me as defining the true character of mathematics, would have appreciated this little book and heartily endorsed its philosophy. This book proffers the thesis that mathematics is actually an easy subject and many of the famous problems, even those in number theory itself, which have famously difficult solutions, can be resolved in simple and more direct terms.

There is no doubt a certain presumptuousness in this claim. The great mathematicians of yesteryear, those working in number theory and related fields, did not necessarily strive to effect the simple solution. They may have felt that the status and importance of mathematics as an intellectual discipline entailed, perhaps indeed required, a weighty solution. Gauss was certainly a wordy master and Euler another. They belonged to a tradition that undoubtedly revered mathematics, but as a discipline at some considerable remove from the commonplace. In keeping with a more democratic concept of intelligence itself, contemporary mathematics diverges from this somewhat elitist view. The simple approach implies a mathematics generally available even to those who have not been favored with the natural endowments, nor the careful cultivation of an Euler or Gauss.

Such an attitude might prove an effective antidote to a generally declining interest in pure mathematics. But it is not so much as incentive that we proffer what might best be called “the fun and games” approach to mathematics, but as a revelation of its true nature. The insistence on simplicity asserts a mathematics that is both “magical” and coherent. The solution that strives to master these qualities restores to mathematics that element of adventure that has always supplied its peculiar excitement. That adventure is intrinsic to even the most elementary description of analytic number theory.

The initial step in the investigation of a number theoretic item is the formulation of “the generating function”. This formulation inevitably moves us away from the designated subject to a consideration of complex variables. Having wandered away from our subject, it becomes necessary to effect a return. Toward this end “The Cauchy Integral” proves to be an indispensable tool. Yet it leads us, inevitably, further afield from all the intricacies of contour integration and they, in turn entail the familiar processes, the deformation and estimation of these contour integrals.

Retracing our steps we find that we have gone from number theory to function theory, and back again. The journey seems circuitous, yet in its wake a pattern is revealed that implies a mathematics deeply inter-connected and cohesive.