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Algebraic Number Theory

Translated from the German
by Norbert Schappacher

With 16 Figures

Springer

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