

APPLIED OPTIMIZATION

Yurii Nesterov

**INTRODUCTORY
LECTURES ON
CONVEX
OPTIMIZATION**

A Basic Course

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