

Jaroslav Nešetřil Vojtěch Rödl (Eds.)

Mathematics of Ramsey Theory

With 15 Figures



Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo
Hong Kong Barcelona

Jaroslav Nešetřil

Department of Applied Mathematics
Charles University
Malostranské nám. 25
11800 Praha 1
Czechoslovakia

Vojtěch Rödl

Department of Mathematics
and Computer Science
Emory University
Atlanta, GA 30322
USA

Mathematics Subject Classification (1980): 00-02, 00A10, 03F30, 05A99,
05B25, 05C15, 05C55, 05C65, 05C80, 11B13, 11B25, 28D99, 34C35,
51E10, 51E15, 51M99, 54H20, 68R10

ISBN-13:978-3-642-72907-2 e-ISBN-13:978-3-642-72905-8
DOI: 10.1007/978-3-642-72905-8

Library of Congress Cataloging-in-Publication Data

Mathematics of Ramsey theory / J. Nešetřil, V. Rödl (eds.) p. cm. - (Algorithms and combinatorics; 5)

Includes bibliographical references (p.) and indexes.

ISBN-13:978-3-642-72907-2

1. Ramsey theory. I. Nešetřil, J. (Jaroslav) II. Rödl, V. (Vojtěch), 1949-. III. Series.
QA166.M37 1990 511'.5-dec20 90-10186 CIP

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. Duplication on this publication or parts thereof is only permitted under the provisions of the German Copyright Law of September 9, 1965, in its current version, and a copyright fee must always be paid. Violations fall under the prosecution act of the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1990
Softcover reprint of the hardcover 1st edition 1990

2141/3140-543210 - Printed on acid-free paper

Table of Contents

Introduction

Ramsey Theory Old and New	1
<i>J. Nešetřil and V. Rödl</i>	

1. Ramsey Numbers	2
2. Transfinite Ramsey Theory	3
3. Chromatic Number	3
4. Classical Theorems	3
5. Other Classical Theorems	4
6. Structural Generalizations	5
7. Infinite Ramsey Theorem	6
8. Unprovability Results	6
9. Non-Standard Applications	7

Part I. Classics

Problems and Results on Graphs and Hypergraphs: Similarities and Differences	12
<i>P. Erdős</i>	

1. Extremal Problems of Turán Type	13
2. Density Problems	15
3. Ramsey's Theorem	17
4. Ramsey-Turán Type Problems	22
5. Chromatic Numbers	25

Note on Canonical Partitions	29
<i>R. Rado</i>	

Part II. Numbers

On Size Ramsey Number of Paths, Trees and Circuits. II	34
<i>J. Beck</i>	

1. Introduction	34
2. Proof of Theorem 1 – Part One	36
3. Proof of Theorem 1 – Part Two	40
4. Proof of Theorem 2	42
5. Proof of Theorem 3	44

On the Computational Complexity of Ramsey-Type Problems	46
<i>S.A. Burr</i>	

1. Introduction	46
2. <i>NP</i> -Complete Ramsey Problems	47
3. Polynomial-Bounded Ramsey Problems	49
4. Discussion	51

Constructive Ramsey Bounds and Intersection Theorems for Sets	53
<i>P. Frankl</i>	

1. Introduction	53
2. Families of Sets with Prescribed Intersections	54
3. The Proof of Theorem 2.3	55
4. The Proof of Theorem 1.1	55

Ordinal Types in Ramsey Theory and Well-Partial-Ordering Theory	57
<i>I. Kríž and R. Thomas</i>	

1. Introduction	58
2. Sheaves	61
3. Ramsey Systems	64
4. Well-Partial-Ordering	71
5. Erdős-Szekeres Theorem	77
6. Ramsey Systems	85
7. Canonical Ramsey Theorem	92

Part III. Structural Theory

Partite Construction and Ramsey Space Systems	98
<i>J. Nešetřil and V. Rödl</i>	

1. Introduction	98
2. Statement of Results	100
3. Partite Lemma	101
4. Partite Construction	106
5. Applications	108

Graham-Rothschild Parameter Sets	113
<i>H.J. Prömel and B. Voigt</i>	
1. Introduction	113
2. Parameter Sets and Parameter Words (Definition and Basic Examples)	114
3. Hales-Jewett's Theorem	117
4. Graham-Rothschild's Theorem	128
5. Infinite Versions	136
6. Other Structures	143
Shelah's Proof of the Hales-Jewett Theorem	150
<i>A. Nilli</i>	
 Part IV. Noncombinatorial Methods	
Partitioning Topological Spaces	154
<i>W. Weiss</i>	
1. Introduction	154
2. Partitioning Singletons	155
3. Partitioning Pairs	167
4. Open Problems	169
Topological Ramsey Theory	172
<i>T.J. Carlson and S.G. Simpson</i>	
1. Introduction	172
2. Ramsey Spaces and Ellentuck's Theorem	173
3. Finitary Consequences of Ellentuck Type Theorems	176
4. The Axiom of Choice and the Construction of Non-Ramsey Sets	177
5. Finite Dimensional Analogues of Ellentuck Type Theorems	179
6. Canonical Partitions	180
Ergodic Theory and Configurations in Sets of Positive Density	184
<i>H. Fürstenberg, Y. Katznelson and B. Weiss</i>	
1. Introduction	184
2. Correspondence Between Subsets of \mathbb{R}^2 and \mathbb{R}^2 -Actions	185
3. Ergodic Averages for Subsets of \mathbb{R}^2	187
4. First Application to Subsets of Positive Density in \mathbb{R}^2	188
5. Proof of Theorem A	189
6. A Recurrence Property of \mathbb{R}^2 -Actions	191
7. Proof of Theorem B	197

Part V. Variations and Applications

Topics in Euclidean Ramsey Theory	200
<i>R.L. Graham</i>	
1. Introduction	200
2. Preliminaries	200
3. Ramsey Sets	202
4. Sphere-Ramsey Sets	208
5. Concluding Remarks	211
On Pisier Type Problems and Results	
(Combinatorial Applications to Number Theory)	214
<i>P. Erdős, J. Nešetřil and V. Rödl</i>	
1. Introduction	214
2. Multiplicative Bases and Szemerédi-Ruzsa Theorem	217
3. Graphical Sequences and Examples of Their Use	221
4. Pisier Type Theorems	224
5. Pisier Problem – Positive Results	227
6. Special Ramsey Graphs – the Partite Construction	228
Combinatorial Statements Independent of Arithmetic	
<i>J. Paris</i>	232
1. Introduction	232
2. Notation	233
3. Arithmetic	235
4. Conclusions	244
Boolean Complexity and Ramsey Theorems	
<i>P. Pudlák</i>	246
1. General Remarks	246
2. An Example of a Lower Bound to Formula Size Complexity	248
Uncrowded Graphs	
<i>J. Spencer</i>	253
1. Graphs	253
2. Hypergraphs	258
3. Heilbronn's Conjecture	259
Author Index	
<i></i>	263
Subject Index	
<i></i>	267