Lecture Notes in Mathematics

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The Adjoint of a Semigroup of Linear Operators

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Preface

This lecture note is an extended version of the author's Ph.D. thesis "The adjoint of a semigroup of linear operators" (Leiden, 1992). The main difference consists of two new chapters (3 and 4) dealing with Hille-Yosida operators, extra- and interpolation and perturbation theory. Also, the sections 7.4 and 8.2 are new.

The general theory of adjoint semigroups was initiated by Phillips [Ph2], whose results are presented in somewhat more generality in the book of Hille and Phillips [HPh], and was taken up a little later by de Leeuw [dL]. Before that, Feller [Fe] had already used adjoint semigroups in the theory of partial differential equations. After these papers almost no new results on adjoint semigroups were published, although the theory of strongly continuous semigroups continued to develop rapidly. Recently the interest in adjoint semigroups revived however, due to many applications that were found to, e.g., elliptic partial differential equations [Am], population dynamics [Ceal-6], [DGT], [GH], [GW], [In], control theory [Heij], approximation theory [Ti], and delay equations [D], [DV], [HV], [V]. This stimulated also renewed interest in the abstract theory of adjoint semigroups, e.g. [Pal-3], [GNa] and [DGH].

The aim of the present lecture note is to give a systematic exposition of the abstract theory of adjoint semigroups. Although we illustrate many results with concrete examples, we do not give applications of the theory. An exposition of the various fields where adjoint semigroups have found fruitful application would require a volume of at least comparable size. Rather, this lecture note should provide the interested reader with sufficient background material in order to make these applications easily accessible.

From the duality relation $\langle T^*(t)x^*, x\rangle = \langle x^*, T(t)x\rangle$, it follows that theorems on C_0-semigroups trivially translate into theorems on their adjoints, the difference being that the weak*-topology of X^* takes over the role of the strong topology of X. For example, $T^*(t)$ is a weak*-continuous semigroup, but not necessarily strongly continuous. From this point of view adjoint semigroups mirror, in a rather bad sense, the properties of their pre-adjoints, and no interesting new phenomena occur. This is, however, not the only way to look at adjoint semigroups. Instead, in this lecture note we try to understand the reasons why the adjoint semigroup fails to be strongly continuous and to study the extent to which it does so, and how this depends on the structure of the underlying Banach space and the properties of the pre-adjoint semigroup.

Roughly speaking, the book consists of two parts. The first part, Chapters

1-5, contains the general theory of adjoint semigroups, whereas the second part, Chapters 6-8, deals with more 'structure theoretical' topics. Let us describe in some more detail the contents of each chapter. In Chapter 1, the basic properties of the adjoint semigroup $T^*(t)$ are proved and the canonical spaces $X^{\odot}$ and $X^{\odot\odot}$ associated with the adjoint semigroup are introduced. Already at this stage we treat the adjoints of certain semigroups arising in a natural way in connection with Schauder bases. The reason is the usefulness of these semigroups for providing counter-examples to many questions in later chapters. In Chapter 2, the $\sigma(X, X^{\odot})$-topology is studied in detail. Many results show that this topology behaves rather like the weak topology, although there are also some differences. We give very simple proof of de Pagter's refinement of Phillips's characterization of $\odot$-reflexivity. In Chapter 3 we start with a systematic study of extrapolation spaces associated to a Hille-Yosida operator, having in mind that A^*, the adjoint of a generator A, is a Hille-Yosida operator. The fact that a Hille-Yosida operator on X extends to a generator of a C_0-semigroup on a suitable extrapolation space X_{-1}, provides us with a very useful tool. It allows a reduction of questions about Hille-Yosida operators and other objects associated to it to the semigroup case. This idea is at the basis of our presentation of perturbation theory in Chapter 4. Performing calculations in X_{-1} rather than in X, simplifies many arguments and reduces the proofs of the various variation-of-constants formulas to trivialities. Also in Chapter 4, we apply these ideas to the study of abstract Cauchy problems and of certain weak*-continuous semigroups on dual spaces. In Chapter 5, we take a closer look to the extent an orbit of the adjoint semigroup can fail to be strongly continuous. If $X^{\otimes}$ denotes the closed subspace of X^* consisting of those elements whose orbits are strongly continuous for $t > 0$, then we show that the quotient space $X^*/X^{\otimes}$ is either zero or non-separable. A modification of the proof is used to show that orbits in the quotient space $X^*/X^{\odot}$ are either identically zero for $t > 0$, or non-separable.

In the last three chapters, we study the relationship between the geometry of the underlying Banach space and the behaviour of the adjoint semigroup, and we take a look at several special classes of semigroups. In Chapter 6, after proving a Hahn-Banach type theorem and giving some applications, we show that there are a number of connections between continuity of the adjoint semigroup and the Banach space X or X^* having the Radon-Nikodym property or not. For example, if X^* has the RNP, then $T^*(t)$ is strongly continuous for $t > 0$. In Chapter 7, which is based on joint work with Günther Greiner, we study the rather delicate problem to describe the semigroup dual of a tensor product of two semigroups in terms of the semigroup duals of the two semigroups. The special case where $T(t)$ is translation with respect to the first coordinate on $C_0(\mathbb{R} \times K)$, is discussed in detail. Finally, in Chapter 8, which is partly based on joint work with Ben de Pagter, we study adjoints of positive semigroups. The problem of determining when the semigroup dual $X^{\odot}$ is a sublattice of X^* is discussed. Although, in general, this problem is difficult, there is detailed information on the behaviour of the adjoint semigroup in the case where X is a $C(K)$-space or $T(t)$ is a multiplication semigroup. Also, there is a section providing semigroup versions of a classical result of Wiener and Young that, with respect to the translation semigroup on $C_0(\mathbb{R})$, $T^*(t)\mu \perp \mu$ for almost all t, if μ is singular with respect to the Lebesgue measure.

At this point I would like to thank a number of persons, who have in one way or another contributed essentially to the present book. First of all, my promotor Odo Diekmann, who always encouraged me to develop my own mathematical interests. I very much appreciate the freedom I experienced in working with him. Also I thank Ben de Pagter for his constant interest and the many stimulating discussions I had with him. Not only does this book contain a number of results due to him or obtained in joint work, also many of my own results can be traced back to ideas of Ben in the form of conjectures or suggestions about what would be an interesting topic to take a closer look at. In particular, this applies to Chapters 5 and 8. I would like to thank the Tübinger school for their hospitality during my half-year stay in the Wintersemester 1990/91. Especially I thank Rainer Nagel and Günther Greiner who always showed much stimulating interest in my work and gave valuable advice. The material of Chapter 7 is joint work with Günther, which was done while he visited the CWI in 1990. Also, during my stay in Tübingen I enjoyed working with him very much. The suggestion to use extrapolation theory in matters related to Hille-Yosida operators, is due to Rainer and turned out unexpectedly useful. The warm and personal way Rainer deals with his students and co-workers is really admirable. Finally, I would like to thank Hans Heesterbeek, whose high spirits and humour made it a pleasure to share a room with him during the past four years, Adri Olde Daalhuis for his TeXnical assistance and, of course, my dear Ele.

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