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(continued following index)

Jean-Claude Nédélec

Acoustic and Electromagnetic Equations

Integral Representations
for Harmonic Problems



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Preface

This book is devoted to the study of the acoustic wave equation and of the Maxwell system, the two most common wave equations encountered in physics or in engineering. The main goal is to present a detailed analysis of their mathematical and physical properties.

Wave equations are time dependent. However, use of the Fourier transform reduces their study to that of harmonic systems: the harmonic Helmholtz equation, in the case of the acoustic equation, or the harmonic Maxwell system. This book concentrates on the study of these harmonic problems, which are a first step toward the study of more general time-dependent problems.

In each case, we give a mathematical setting that allows us to prove existence and uniqueness theorems. We have systematically chosen the use of variational formulations related to considerations of physical energy.

We study the integral representations of the solutions. These representations yield several integral equations. We analyze their essential properties. We introduce variational formulations for these integral equations, which are the basis of most numerical approximations.

Different parts of this book were taught for at least ten years by the author at the post-graduate level at Ecole Polytechnique and the University of Paris 6, to students in applied mathematics. The actual presentation has been tested on them. I wish to thank them for their active and constructive participation, which has been extremely useful, and I apologize for forcing them to learn some geometry of surfaces.

A large part of the material contained in this book would not have been in the present state without the work of my many students. Among the

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I also want to thank the persons who were always my main support, my wife, Henriette, especially for her patience, and my children, Laurence and François.

Palaiseau, France

Jean-Claude Nédélec
February 2000

Contents

Preface	v
1 Some Wave Equations	1
1.1 Introduction	1
1.2 Physical Background	3
1.2.1 The acoustic equation	3
1.2.2 The Maxwell equations	4
1.2.3 Elastic waves	6
2 The Helmholtz Equation	9
2.1 Introduction	9
2.2 Harmonic Solutions	10
2.3 Fundamental Solutions	12
2.4 The Case of the Sphere in \mathbb{R}^3	13
2.4.1 Spherical harmonics	15
2.4.2 Legendre polynomials	17
2.4.3 Associated Legendre functions	22
2.4.4 Vectorial spherical harmonics	35
2.5 The Laplace Equation in \mathbb{R}^3	40
2.5.1 The sphere	40
2.5.2 Surfaces and Sobolev spaces	47
2.5.3 Interior problems: Variational formulations	55
2.5.4 Exterior problems	59
2.5.5 Regularity properties of solutions in \mathbb{R}^n	65

2.5.6	Elementary differential geometry	67
2.5.7	Regularity properties	79
2.6	The Helmholtz Equation in \mathbb{R}^3	84
2.6.1	The spherical Bessel functions	85
2.6.2	Dirichlet and Neumann problems for a sphere . .	90
2.6.3	The capacity operator T	97
2.6.4	The case of a plane wave	97
2.6.5	The exterior problem for the Helmholtz equation	102
3	Integral Representations and Integral Equations	110
3.1	Integral Representations	110
3.2	Integral Equations for Helmholtz Problems	117
3.2.1	Equations for the single layer potential	117
3.2.2	Equations for the double layer potential	119
3.2.3	The spherical case	120
3.2.4	The far field	122
3.2.5	The physical optics approximation for the sphere	130
3.3	Integral Equations for the Laplace Problem	135
3.4	Variational Formulations for the Helmholtz Problems . .	141
3.4.1	The operator S	141
3.4.2	Fredholm operators	142
3.4.3	The operator N	143
3.4.4	Formulation with the far field	146
4	Singular Integral Operators	150
4.1	The Hilbert Transform	150
4.2	Singular Integral Operators in \mathbb{R}^n	157
4.2.1	Odd kernels	158
4.2.2	The M. Riesz transforms	160
4.2.3	Adjoint operators	166
4.3	Application to Integral Equations	167
4.3.1	Introduction	167
4.3.2	Homogeneous kernels	168
4.3.3	Pseudo-homogeneous kernels	173
4.4	Application to Integral Equations	175
5	Maxwell Equations and Electromagnetic Waves	177
5.1	Introduction	177
5.2	Fundamental Solution and Radiation Conditions	179
5.3	Multipole Solutions	185
5.3.1	Multipoles	185
5.3.2	The capacity operator	200

5.4	Exterior Problems	204
5.4.1	Trace and lifting associated with the space $H(\text{curl})$	205
5.4.2	Variational formulations for the perfect conductor problem	214
5.4.3	Coupled variational formulations for impedance conditions	226
5.5	Integral Representations	234
5.6	Integral Equations	243
5.6.1	The perfect conductor	243
5.6.2	The zero frequency limit	250
5.6.3	The dielectric case	253
5.6.4	The infinite conductivity limit: The perfect conductor	266
5.7	The Far Field	288
5.7.1	Far field and scattering amplitude	288
5.7.2	Integral equations and far field	298
	References	301
	Index	313