

---

# Proximity Approach to Problems in Topology and Analysis

---

von  
Somashekhar Naimpally

---

Oldenbourg Verlag München

---

---

Somashekhar Nainpally is an Emeritus Professor at Lakehead University, Thunder Bay, Canada. He studied at the University of Bombay, Mumbai, India and Michigan State University, East Lansing, USA. He has coauthored books on Proximity Spaces, Symmetric Generalized Topological Structures and Leelavati.

© 2009 Oldenbourg Wissenschaftsverlag GmbH  
Rosenheimer Straße 145, D-81671 München  
Telefon: (089) 45051-0  
oldenbourg.de

All rights reserved; no part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise without the prior written permission of the Publishers.

Editor: Kathrin Mönch  
Producer: Dr. Rolf Jäger  
Cover design: Kochan & Partner, München  
Printed on acid-free and chlorine-free paper  
Printing: Books on Demand GmbH, Norderstedt

ISBN 978-3-486-58917-7

# Contents

<b>Preface</b>	<b>V</b>
<b>Foreword</b>	<b>VII</b>
<b>Intuitive Introduction</b>	<b>1</b>
<b>1 The Lodato Proximity</b>	<b>3</b>
1.1 Introduction .....	4
1.2 Tools: Clusters and Bunches .....	13
1.3 A General $T_1$ Compactification .....	16
1.4 Mozzochi Uniformity and LO-Proximity .....	21
<b>2 The Efremovič Proximity</b>	<b>25</b>
2.1 Introduction .....	25
2.2 Tools: Clusters .....	29
2.3 Smirnov Compactification .....	31
2.4 Tychonoff Topology, Efremovič Proximity and Uniformity .....	33
2.5 Appendix: Application of Wallman-Frink Compactification .....	36
<b>3 Extensions of Continuous Functions from Dense Subspaces</b>	<b>39</b>
3.1 Generalized Taimanov Theorem .....	39
3.2 Hausdorff Compactifications .....	42
3.3 Applications of the Generalized Taimanov Theorem .....	43
3.4 More Applications .....	46
<b>4 Nearness and Contiguity. Extension Theorems</b>	<b>49</b>
4.1 Introduction .....	50
4.2 Extension Theorems .....	53
4.3 Topological Properties in Terms of Nearness .....	56

4.4	Categorical Point of View.....	58
<b>5</b>	<b>Hyperspace Topologies</b>	<b>61</b>
5.1	What Is a Hit-and-Miss Topology? An Intuitive Introduction.....	61
5.2	Vietoris Proximal and Fell Topologies.....	64
5.3	Hausdorff Metric Topology.....	67
5.4	Wijsman Convergence Topology.....	70
5.5	Locally Finite Topology.....	73
5.6	Ball and Proximal Ball Topologies.....	75
5.7	Poppe's Delta Topologies.....	78
5.8	Hausdorff-Bourbaki Uniformity.....	79
5.9	Uniformly Discrete Hypertopology.....	83
5.10	U Topology.....	85
5.11	Bounded Topologies.....	85
<b>6</b>	<b>Hit-And-Far-Miss Topology: A Generalization of the Wijsman Topology</b>	<b>87</b>
6.1	Introduction.....	87
6.2	Comparisons among the three topologies.....	88
6.3	Comparisons with Other Topologies.....	91
6.4	Appendix I: Relative Total Boundedness.....	92
6.5	Appendix II: Bombay Topology, a Unification.....	93
<b>7</b>	<b>Infimum and Supremum of Hypertopologies</b>	<b>97</b>
7.1	Inf of Hausdorff-Bourbaki Uniformities.....	97
7.2	Sup of Hausdorff-Bourbaki Uniformities.....	99
7.3	Sup of Wijsman Topologies.....	100
<b>8</b>	<b>Hyper-Continuous Convergence in Function Spaces</b>	<b>103</b>
8.1	Introduction.....	103
8.2	Hausdorff-Continuous Convergence.....	105
8.3	Vietoris convergence.....	108
8.4	Proximal-Continuous-Convergence.....	109
8.5	Fell-Continuous-Convergence.....	110

<b>9</b>	<b>Proximal Set-Open Topology</b>	<b>113</b>
9.1	Introduction .....	113
9.2	Comparison of Proximal Set-Open Topology with Other Topologies .....	118
9.3	Separation Axioms .....	118
<b>10</b>	<b>A Unified Approach to Metrisation Problems</b>	<b>121</b>
<b>11</b>	<b>Semi-Metric and Developable Spaces</b>	<b>125</b>
11.1	Introduction .....	125
11.2	Developable Spaces.....	127
11.3	Metrisable Spaces.....	129
11.4	Mappings.....	130
<b>12</b>	<b>Open and Uniformly Open Relations</b>	<b>133</b>
<b>13</b>	<b>Duality in Function Spaces</b>	<b>139</b>
13.1	Introduction .....	139
13.2	Equicontinuity .....	143
13.3	Even Continuity.....	144
13.4	Topological Groups of Homeomorphisms .....	147
13.5	Generalizations of Equicontinuity .....	151
13.6	Application.....	153
<b>14</b>	<b>Proximal and Other Characterizations of Uniform Invariants</b>	<b>155</b>
14.1	Completeness .....	155
14.2	Total Boundedness .....	157
14.3	UC Spaces .....	158
14.4	CU Spaces .....	160
14.5	Compactness.....	160
<b>15</b>	<b>Applications</b>	<b>163</b>
15.1	General Relativity .....	163
References	.....	166
15.2	Differential Equations and Mathematical Economics .....	167
References	.....	170
15.3	Digital Images and Sound Analysis & Synthesis .....	171
References	.....	176

---

<b>Bibliography</b>	<b>177</b>
References for quasi-proximities.....	192
Basic facts .....	192
Cardinality of compatible quasi-proximities and of a quasi-proximity class .....	194
Completions and compactifications .....	196
Hypertopologies and functions spaces .....	198
Fuzzy topology.....	199
<b>Index</b>	<b>201</b>