

George W. Hart

Multidimensional Analysis

Algebras and Systems for
Science and Engineering

With 19 illustrations



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