

Ingo Müller Tommaso Ruggeri

Extended Thermodynamics

With 43 Illustrations



Springer-Verlag

New York Berlin Heidelberg London Paris
Tokyo Hong Kong Barcelona Budapest

Ingo Müller
Fachbereich 10
Verfahrenstechnik und Energietechnik
Technische Universität Berlin
Straße des 17. Juni 135
1000 Berlin 12
Germany

Tommaso Ruggeri
Università degli Studi di Bologna
C.I.R.A.M.
Centro Interdipartimentale di Ricerca
per le Applicazioni della Matematica
Via Saragozza, 8
40123 Bologna
Italy

Mathematics Subject Classification (1991): 35L65, 82C35, 82C40

Library of Congress Cataloging-in-Publication Data

Müller, Ingo, 1936–

Extended thermodynamics/Ingo Müller & Tommaso Ruggeri.

p. cm. — (Springer tracts in natural philosophy; v. 37)

Includes bibliographical references.

ISBN-13: 978-1-4684-0449-4

I. Thermodynamics. I. Ruggeri, Tommaso. II. Title.

III. Series.

QC311.M783 1993

536'.7—dc20

92-26754

Printed on acid-free paper.

© 1993 Springer-Verlag New York, Inc.

Softcover reprint of the hardcover 1st edition 1993

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Production managed by Francine Sikorski; manufacturing supervised by Vincent Scelta.

Photocomposed copy prepared using LaTeX.

ISBN-13: 978-1-4684-0449-4

e-ISBN-13: 978-1-4684-0447-0

DOI: 10.1007/978-1-4684-0447-0

Springer Tracts in Natural
Philosophy

Volume 37

Edited by C. Truesdell

Springer Tracts in Natural Philosophy

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Preface

Physicists firmly believe that the differential equations of nature should be hyperbolic so as to exclude action at a distance; yet, the equations of irreversible thermodynamics—those of Navier–Stokes and Fourier—are parabolic. This incompatibility between the expectation of the physicist and the classical laws of thermodynamics has prompted the formulation of extended thermodynamics.

The main ingredients of extended thermodynamics are

- field equations of balance type,
- constitutive quantities depending on the present local state, and
- entropy as a concave function of the state variables.

This set of assumptions leads to first-order quasi-linear symmetric hyperbolic field equations, guarantees the well-posedness of initial value problems, and permits the treatment of shock waves.

Several tenets of irreversible thermodynamics had to be changed in subtle ways to make extended thermodynamics work. Thus, the entropy is allowed to depend on nonequilibrium variables, the entropy flux is a general constitutive quantity, and the equations for stress and heat flux contain inertial terms. New insight is therefore provided into the principle of material frame indifference.

With these modifications an elegant formal structure can be set up in which, just as in classical thermostatics, all restrictive conditions—derived from the entropy principle—take the form of integrability conditions.

Also the modifications made by extended thermodynamics render the theory fully consistent with the kinetic theory of gases, in particular, Grad’s 13-moment version of the kinetic theory of gases. In fact, extended thermodynamics is most restrictive for gases or, more generally, for bodies whose constituent particles have large mean free paths. Most of this book, therefore, deals with gases: classical ideal gases, degenerate gases, relativistic gases, and mixtures of gases. It puts into perspective the various phenomena called *second sound*, viz. heat propagation, propagation of shear stress, and the second sound in superfluid helium.

Phonons and photons may have large mean free paths as well, and therefore, they are amenable to a treatment by extended thermodynamics. Two brief chapters describe the present status of the systematic theory in this field, which is still progressing.

A certain disappointment with extended thermodynamics of 13 or 14 fields is created by the observation that it describes resonance experiments and light scattering data only slightly better than the conventional theory. These data require further extensions to many fields, as is demonstrated in the last chapters of the book.

We may now say that extended thermodynamics provides a hierarchy of symmetric hyperbolic equations that are relevant for the solutions of initial and boundary value problems provided that the frequencies and wave numbers of their Fourier spectra do not exceed the appropriate limits.

Berlin, Germany
Bologna, Italy

Ingo Müller
Tommaso Ruggeri

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