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Fuzzy Group Theory

Springer

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Preface

Lotfi A. Zadeh introduced the notion of a fuzzy subset of a set in his paper published in 1965. Zadeh's ideas marked a new direction and stirred the interest of researchers worldwide. It provided tools and an approach to model imprecision and uncertainty present in phenomena that do not have sharp boundaries. Rapid theoretical developments and practical applications based on the concept of a fuzzy subset were seen to emerge soon after that.

In 1971, Azriel Rosenfeld used the notion of a fuzzy subset of a set to introduce the notion of a fuzzy subgroup of a group. Rosenfeld's paper inspired the development of fuzzy abstract algebra. He also introduced fuzzy graphs, an area which has been growing actively since then. This is the first book dedicated entirely to the rapidly growing field of fuzzy group theory. It is not easy to present in a single 300 page book all that has been done in fuzzy group theory up-to-date. However, the authors have made a sincere effort to present in a systematic way some results that have appeared in papers and conference proceedings (including some work by the authors themselves). We thank the researchers worldwide for their contributions to this growing field and special thanks to those whose work is referenced in our book. We hope that the reader will find this book crisp and not fuzzy in presentation, as well as rewarding and motivating for developing further results and applications of fuzzy group theory. The material presented in this book has been selected so as to make this a good reference for graduate students and researchers working in fuzzy group theory. The end of each chapter lists numerous references. While some of those have contributed to the material in the chapters, others are directly related to the material presented and so have been listed there.

In Chapter 1, we first present some basic material concerning fuzzy subsets of a set. We then introduce the notion of a fuzzy subgroup of a group and develop some concepts such as normal fuzzy subgroups and complete and weak direct products of fuzzy subgroups. We also present the notion of the fuzzy order of an element of a group.

In Chapter 2, we present several fuzzy versions of Lagrange's Theorem and Caley's Theorem. We consider fuzzy quotient groups, characteristic fuzzy subgroups and conjugate fuzzy subgroups.

The notion of the ascending central series of a fuzzy subgroup is presented in Chapter 3 and used to define nilpotency of a fuzzy subgroup. The notion of the descending central series of a fuzzy subgroup is also presented and used to define the nilpotency of a fuzzy subgroup. It is shown that these two definitions are not equivalent. The notion of commutators to generate the derived chain of a fuzzy subgroup is introduced and is used to define a solvable fuzzy subgroup. Fuzzy versions of well-known crisp results are presented in this chapter.

Fuzzy subgroups of Hamiltonian, solvable, P-Hall, and nilpotent groups are examined in Chapter 4. The notions of generalized characteristic fuzzy subgroups, fully invariant fuzzy subgroups, and characteristic fuzzy subgroups are introduced. It is shown that if G is a finite group all of whose Sylow subgroups are cyclic, then a fuzzy subgroup of a group G is normal if and only if it is a generalized fuzzy subgroup of G. Normal fuzzy subgroups, quasi-normal fuzzy subgroups, (p, q)-subgroups, fuzzy cosets, fuzzy conjugates and $SL(p, q)$-subgroups are also considered in this chapter.

In Chapter 5, we present two approaches to show the existence of free fuzzy subgroups. One features the approach by Garzon and Muganda.

In Chapter 6, we study fuzzy subgroups of Abelian groups. We develop the notions of independent generators, primary fuzzy subgroups, divisible fuzzy subgroups, and pure fuzzy subgroups. We determine a complete system of invariants for those fuzzy subgroups which are direct sums of fuzzy subgroups whose supports are cyclic. We also develop the notions of basic fuzzy subgroups and p-basic fuzzy subgroups.

In Chapter 7, we introduce the notion of the fuzzy direct product of fuzzy subgroups defined over subgroups of a group. These ideas are applied to the problem in group theory of obtaining conditions under which a group G can be expressed as the direct product of its normal subgroups.

The number of fuzzy subgroups of certain finite Abelian groups with respect to a suitable equivalence relation as determined by Murali and Makamba are considered in Chapter 8. The Abelian groups under consideration are those which are direct sums of cyclic groups of prime order and those of order $p^n q^m$ for distinct primes p and q and nonnegative integers n and m.

In Chapter 9, we present the work of Tom Head concerning methods for deriving fuzzy theorems from crisp ones and embedding lattices of fuzzy subgroups into lattices of crisp subgroups. We also present the work of Ajmaal and Thomas concerning properties of lattices of fuzzy subgroups.

The first part of Chapter 10 is concerned with deriving membership functions from similarity relations. An algebraic approach for the construction of fuzzy subgroups is also considered. The chapter closes with some applications of fuzzy subgroups to a generalized recognition problem.

We have done our best to provide the reader with a complete bibliography used for writing this book. We welcome comments and suggestions by the readers and apologize in advance if we inadvertently missed any source of reference in our bibliography.

The authors are grateful to the staffs of Springer-Verlag, especially Frank Holzwarth, Gabriele Maas, Heather King, Janusz Kacprzyk and Dr. Thomas Ditzinger. We are indebted to Dr. Timothy Austin, Dean, Creighton College of Arts and Sciences and to Dr. and Mrs. George Haddix for their support of our work. We also wish to thank Professor Paul Wang of Duke University for his strong support of fuzzy mathematics. The first author dedicates the book to his grandchildren, Emily and twins Emma and Jenna. The second author dedicates the book to her supportive husband Ravi, and loving sons Navin and Manoj. Together, we dedicate this book to the family of Professor Azriel Rosenfeld.

John Mordeson
Creighton University

Kiran R. Bhutani
The Catholic University of America

Azriel Rosenfeld
University of Maryland

We were deeply saddened by the passing away of Professor Rosenfeld in February 2004. His presence would have added further strength to our book. We have lost a great collaborator, an outstanding scientist and a wonderful mentor. He will be missed sorely.

John Mordeson
Kiran R. Bhutani

Azriel Rosenfeld (1931-2004)

Azriel Rosenfeld was a tenured research Professor, a Distinguished University Professor (since 1995), and founder (1983) and Director of the Center for Automation Research, a department level unit of the College of Computer, Mathematical and Physical Sciences at the University of Maryland in College Park. He directed the Center until his retirement in June 2001. Upon his retirement he was designated a Distinguished University Professor Emeritus in the University's Institute for Advanced Studies. He also held affiliate professorships in the Departments of Computer Science and Psychology and in the College of Engineering. He held a Ph. D. in mathematics from Columbia University (1957), rabbinic ordination (1952) and a Doctor of Hebrew Literature degree (1955) from Yeshiva University and honorary Doctor of Technology degrees from Linkoping University, Sweden (1980) and Oulu University, Finland (1994), and an honorary Doctor of Humane Letters degree from Yeshiva University (2000).

Dr. Rosenfeld was widely regarded as the leading researcher in the world in the field of computer image analysis. Over a period of more than 35 years he made many fundamental and pioneering contributions to nearly every area of that field. He wrote the first textbook in the field (1969): was founding editor of its first journal (1972); and was co-chairman of its first international conference (1987). He published 30 books and over 600 book chapters and journal articles, and directed over 50 Ph. D. dissertations.

He was a Fellow of the Institute of Electrical and Electronics Engineers (1971), and won its Emanuel Piore Award in 1985; he was a founding Fellow of the American Association for Artificial Intelligence (1990) and of the Association for Computing Machinery (1993); he was a Fellow of the Washington Academy of Sciences (1998), and won its Mathematics and Computer Science Award in 1988; he was a founding Director of the Machine Vision Association of the Society of Manufacturing Engineers (1985-8), won its President's Award in 1987, and was a certified Manufacturing Engineer (1988); he was a founding member of the IEEE Computer Society's Technical Committee

on Pattern Analysis and Machine Intelligence (1965), served as its Chairman (1985-7), and received the Society's Meritorious Service Award in 1986; and its Harry Goode Memorial Award in 1944 and became a Golden Core member of the Society in 1996; he received the IEEE Systems, Man, Cybernetics Norbert Wiener Award in 1995; he received an IEEE Standards Medallion in 1990, and the Electronic Imaging International Imager of the Year Award in 1991; he was a founding member of the Governing Board of the International Association for Pattern Recognition (1978-85), served as its President (1980-2), won its first K. S. Fu Award in 1988, and became one of its founding Fellows in 1994; he was a Foreign Member of the Academy of Science of the German Democratic Republic (1988-92), and was a Corresponding Member of the National Academy of Engineering of Mexico (1982).

In 1985, he served as chairman of a panel appointed by the National Research Council to brief the President's Science Advisor on the subject of computer vision; he has also served (1985-8) as a member of the Vision Committee of the National Research Council. In 1982-3 and 1986-8 he served as a member of a task force appointed by the Defense Science Board to review the state of the art in automatic target recognition, and in 1993-4 he chaired a panel that conducted an assessment of foreign pattern recognition and image understanding research and development.

We were deeply saddened by his death on Sunday, February 22, 2004. His love of knowledge, his passion and dedication to research, his generosity and kindness will always be remembered.

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