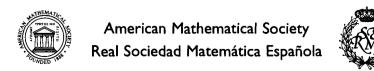
Curves and Surfaces

Sebastián Montiel Antonio Ros

Translated by Sebastián Montiel

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