Series on Advances in Mathematics for Applied Sciences – Vol. 46

M Mokhtar-Kharroubi
Laboratoire de Mathématiques et Mécanique Théorique
Université de Franche-Comté
Besançon
France

Mathematical Topics in Neutron Transport Theory

New Aspects

World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data

Mokhtar-Kharroubi, M.

Mathematical topics in neutron transport theory : new aspects / M. Mokhtar-Kharroubi.

p. cm. -- (Series on advances in mathematics for applied sciences - Vol. 46)

Includes bibliographical references and index.

ISBN 9810228694 (alk. paper)

1. Neutron transport theory -- Mathematics. 2. Mathematical physics. I. Title. II. Series.

QC793.5.N4655M65 1997

621.48'31--dc21 97-21755

CIP

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

Copyright © 1997 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

This book is printed on acid-free paper.

Printed in Singapore by Uto-Print

Contents

1 Introduction 1

2 Compactness properties of perturbed c_0-semigroups 9
2.1 Introduction . . . 9
2.2 On essential type of c_0-semigroups . . . 10
2.3 On the strong convex compactness property . . . 12
2.4 Compactness properties of perturbed semigroups . . . 14
2.5 On the stability of the essential type . . . 21
2.6 Comments . . . 25
References . . . 26

3 Regularity of velocity averages 29
3.1 Introduction . . . 29
3.2 Stationary problems . . . 31
3.3 Evolution problems . . . 39
3.4 Comments . . . 44
References . . . 46

4 Spectral analysis of transport equations. A unified theory 51
4.1 Introduction . . . 51
4.2 Stationary problems . . . 55
4.3 Evolution problems in L^p $(1<p<\infty)$ . . . 65
4.4 Evolution problems in L^1 . . . 77
4.5 The effects of delayed neutrons . . . 85
4.6 Comments . . . 90
References . . . 95

5 On the leading eigenelements of transport operators 99
5.1 Introduction . . . 99
5.2 Spectral properties of positive operators . . . 100

5.3 The irreducibility of transport semigroups 108
5.4 A general existence result 112
5.5 A spectral inequality 114
5.6 Nonexistence results 116
5.7 Existence results 118
5.8 Strict monotonicity properties of the leading eigenvalue . . 124
5.9 Domain derivative of the leading eigenvalue 128
5.10 An approximation theory of the leading eigenelements . . . 130
5.11 The criticality eigenvalue problem 133
5.12 The effects of delayed neutrons 136
5.13 Comments 139
References 141

6 Spectral theory of transport operators with form positive collision operators 145
6.1 Introduction 145
6.2 Self-adjointness and quadratic form 147
6.3 Existence results for given spatial domains 152
6.4 Existence results for large spatial domains 160
6.5 The isotropic models 163
6.6 Comments 168
References 170

7 On Miyadera perturbations of c_0-semigroups 173
7.1 Introduction 173
7.2 A perturbation theorem 174
7.3 The essential type of Miyadera perturbations 180
7.4 Comments 184
References 184

8 On resolvent positive operators and positive c_0-semigroups in $L^1(\mu)$ spaces 185
8.1 Introduction 185
8.2 A preliminary result 186
8.3 Miyadera perturbations in $L^1(\mu)$ spaces 187
8.4 Alternative proof of Desch's theorem 191
8.5 Comments 195
References 195

9 On singular neutron transport equations in L^1 spaces 197
9.1 Introduction 197
9.2 Generation results 199
9.3 The essential type of the perturbed semigroup 203
9.4 Comments 213
References 214

10 Stochastic formulations of neutron transport. Nonlinear problems 215
10.1 Introduction 215
10.2 Preliminary results 217
10.3 Maximal solution of the stationary problem 223
10.4 The subcritical case 225
10.5 The critical case 225
10.6 The supercritical case 226
10.7 On the uniqueness 230
10.8 The time dependent problem 237
10.9 Time asymptotic behaviour 238
10.10Comments 242
References 243

11 Velocity averages and inverse problems 245
11.1 Introduction 245
11.2 One dimensional inverse problems 247
11.3 Multidimensional inverse problems 252
11.4 The dimension two 258
11.5 Characterization of the range of integrated fluxes 260
11.6 Comments 262
References 263

12 Limiting absorption principles and wave operators in $L^1(\mu)$ spaces with applications to transport theory 267
12.1 Introduction 267
12.2 Preliminary results 270
12.3 On the wave operators 275
12.4 The similarity of T_0 and T 277
12.5 Converse results 278
12.6 Scattering theory for transport operators 281
12.7 Comments 286
References 287

13 Lin's factorization formalism and applications to transport theory **291**
13.1 Introduction 291
13.2 A preliminary result 292
13.3 On the wave operator $s\lim_{t\to+\infty} U(t)U_0(-t)$ 294
13.4 Application to transport groups. Part 1 297
13.5 Application to transport groups. Part 2 302
13.6 Comments 306
References 307

14 Inverse scattering and albedo operator. By M. Choulli and P. Stefanov **309**
14.1 Introduction 309
14.2 The special solution and the scattering operator 314
14.3 Reconstruction of σ_a, k from S 320
14.4 The albedo operator 326
14.5 Reconstruction of σ_a, k from \$. The non-convex case 330
References 339

Index **343**