

Elisabeth Bouscaren (Ed.)

# Model Theory and Algebraic Geometry

An introduction  
to E. Hrushovski's proof of the geometric  
Mordell-Lang conjecture



Springer

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# Contents

## Introduction to model theory

ELISABETH BOUSCAREN	1
1 Structures, language associated to a structure . . . . .	1
2 Definable sets and formulas, satisfaction . . . . .	6
3 Theories, elementary substructures, compactness . . . . .	8
4 Types . . . . .	13
References . . . . .	17

## Introduction to stability theory and Morley rank

MARTIN ZIEGLER	19
1 The monster model, imaginary elements . . . . .	19
2 Morley rank . . . . .	23
3 Definable types . . . . .	27
4 Forking . . . . .	29
5 Strongly minimal sets . . . . .	33
6 Orthogonality . . . . .	40
7 Appendix . . . . .	43
References . . . . .	43

## Omega-stable groups

DANIEL LASCAR	45
1 Prerequisites and notation . . . . .	45
2 Chain conditions . . . . .	46
3 Stabilizers . . . . .	48
4 Generic types . . . . .	49
5 The indecomposability theorem . . . . .	52
6 One-based groups . . . . .	54
7 Almost strongly minimal subgroups . . . . .	57
References . . . . .	59

**Model theory of algebraically closed fields**

ANAND PILLAY	<b>61</b>
1 Algebraically closed fields . . . . .	61
2 Zariski closed sets . . . . .	66
3 Varieties . . . . .	70
4 Algebraic groups . . . . .	75
5 $\omega$ -stable fields. . . . .	82
References . . . . .	83

**Introduction to abelian varieties and the Mordell-Lang conjecture**

MARC HINDRY	<b>85</b>
1 Abelian varieties . . . . .	85
2 Lang's conjecture . . . . .	91
3 Diophantine equations over function fields, the "relative" case of Lang's conjecture . . . . .	95
4 Commented bibliography . . . . .	97
References . . . . .	98

**The model-theoretic content of Lang's conjecture**

ANAND PILLAY	<b>101</b>
1 Introduction . . . . .	101
2 Stability and structures of Lang-type . . . . .	101
3 Concluding remarks. . . . .	105
References . . . . .	105

**Zariski geometries**

DAVID MARKER	<b>107</b>
1 Definitions . . . . .	107
2 Preliminary concerns . . . . .	111
3 Ample geometries . . . . .	115
4 Brief remarks on the proofs of Theorem 3.3 . . . . .	117
5 Generalizations . . . . .	119
6 Two examples . . . . .	120
7 Appendix: classical dimension theory . . . . .	124
References . . . . .	127

**Differentially closed fields**

CAROL WOOD	<b>129</b>
1 Notation and basic facts . . . . .	129
2 Types over differential fields . . . . .	133
3 Fields interpretable in differentially closed fields . . . . .	135
4 The Manin kernel . . . . .	137
References . . . . .	141

**Separably closed fields**

FRANÇOISE DELON	<b>143</b>
1 Fields of characteristic $p > 0$ . . . . .	143
2 Separably closed fields. Theories and types . . . . .	146
3 $\lambda$ -closed subsets of affine space . . . . .	153
4 $\lambda$ -closed subsets of a fixed type . . . . .	159
5 $\lambda$ -closed subsets of a minimal type . . . . .	162
6 Thin types . . . . .	174
References . . . . .	175

**Proof of the Mordell-Lang conjecture for function fields**

ELISABETH BOUSCAREN	<b>177</b>
1 The general statement . . . . .	177
2 The characteristic 0 case . . . . .	180
2.1 The model theoretic setting and the dichotomy . . . . .	180
2.2 A simple case as a warm up . . . . .	182
2.3 The general characteristic zero case . . . . .	183
3 The characteristic $p$ case . . . . .	188
References . . . . .	196

**Proof of Manin's theorem by reduction to positive characteristic**

EHUD HRUSHOVSKI	<b>197</b>
References . . . . .	204

**Index****207**