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Partial Differential Equations of Elliptic Type

Second Revised Edition

Translated by Zane C. Motteler

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Preface to the First Edition

In the theory of partial differential equations, the study of elliptic equations occupies a preeminent position, both because of the importance which it assumes for various questions in mathematical physics, and because of the completeness of the results obtained up to the present time.

In spite of this, even in the more classical treatises on analysis the theory of elliptic equations has been considered and illustrated only from particular points of view, while the only expositions of the whole theory, the extremely valuable ones by LICHTENSTEIN and ASCOLI, have the character of encyclopedia articles and date back to many years ago.

Consequently it seemed to me that it would be of some interest to try to give an up-to-date picture of the present state of research in this area in a monograph which, without attaining the dimensions of a treatise, would nevertheless be sufficiently extensive to allow the exposition, in some cases in summary form, of the various techniques used in the study of these equations.

At present the interest of researchers seems to be directed mainly toward the study of mixed problems, of equations of higher order, and of systems of equations. This part of the theory is therefore in a phase of rapid and continual evolution, so that it does not seem that the moment has yet come to give it a systematic exposition. On the other hand, the methods used in this current research do not greatly differ, at least in their general conception, from those that have proved effective in the study of the DIRICHLET, NEUMANN, and oblique derivative problems for a single equation of second order. It therefore seemed to me opportune to dwell primarily on this initial part of the theory, which is the part which has by now reached a sufficiently stable state and whose study is without doubt indispensable to an understanding of the methods used in the most recent research.

Therefore the aspects of the theory to which I have decided to give the most complete treatment are: I. GIRAUD'S method for transforming boundary value problems into integral equations of the second kind. II. The study of boundary value problems by means of various procedures from linear functional analysis which originated in some works by CACCIOPPOLI, PICONE, WEYL. III. The research of BERNSTEIN, HOPF, SCHAUDER, LERAY, and CACCIOPPOLI based on the a priori majorization of the solutions of linear and nonlinear equations.

On the other hand, apart from references, I have not digressed either on the method of minima or on the so-called „kernel function" method, because the classical treatise of COURANT-HILBERT is authoritative on the first, and an exhaustive monograph by BERGMAN and SCHIFFER was recently devoted to the second.

The three aspects of the theory referred to above occupy Chapters III, IV, V, VI, of which the first three are devoted to linear equations, the last to nonlinear ones. On the other hand, Chapter I, in which we summarize various classical notions of a general character, and Chapter II, devoted to the study of generalized potentials, are of a preliminary character. Finally, in Chapter VII various other questions again concerning a single equation of second order or else equations of higher order, systems of equations, or problems depending on a parameter are taken into consideration. This last chapter, in contrast to the first six, and for reasons which were pointed out earlier has essentially the character of a bibliographic summary. The volume ends with an ample Bibliography comprising more than six hundred works, almost all published after 1924. For references to this Bibliography I adopted the convention of numbers in brackets; for the bibliography prior to 1924 I refer the reader to LICHTENSTEIN'S article in the Encyclopedia or else I give some indication in footnotes at the bottom of the pages.

I should add informally that I have been almost exclusively concerned with the study of elliptic equations of general type and have hardly occupied myself at all with equations of special type. Consequently all the research which centres round the theory of harmonic functions, and within that theory the part, however interesting, relating to problems on free boundaries, does not occur in the Bibliography and remains outside the scope of our exposition. I have tried instead to compile the bibliography relating to polyharmonic functions and to the equations of elasticity, and to comment briefly on it. This part of the Bibliography is thus undoubtedly incomplete, because it contains none of the works of a predominantly mathematical-physical character. I cannot end without expressing my keenest gratitude to Springer-Verlag for accepting this book in their series, and for giving it a perfect typographical format.

Naples, 28 February 1954 CARLO MIRANDA

Preface to the Second Edition

When, upon the invitation of Springer Verlag, I began to prepare the second edition of this monograph, I was already conversant with the great progress made on the qualitative plane in the theory of elliptic equations in the fourteen years which had elapsed since the first edition. However, I would never have thought that on the quantitative plane the progress in this branch of analysis had reached that extent which now emerges from a careful examination of the bibliography. Suffice it to say that, while in the Bibliography to the first edition, which refers to the years between 1924 and 1953, somewhat more than 600 works were listed, yet from 1953 until now, namely in a period of time which is half the preceding one, more than 1600 works were published, for which a complete bibliography[1] would comprise more than 2200 items.

It was thus immediately clear that to illustrate this mass of work in a volume of the normal dimensions of an Ergebnisse volume would be entirely impossible and that it would be necessary to give up the treatment of certain topics which could be eliminated without modifying the general plan of the work. In agreement with the Editors of the Ergebnisse, it was therefore decided that in the new edition §§ 57 and 58 would be suppressed, the first because a nice monograph by S. BERGMAN [3] related to this topic has appeared in the meantime, the second because its theme, namely the spectral theory of elliptic equations, has acquired such dimensions that it can only be treated in a satisfactory way in a volume devoted exclusively to it.

By eliminating from the old and new bibliography mention of about 400 works relating to the topics of §§ 57 and 58, as well as many early notes and various expository works which have by now lost all interest, and also by referring to other texts for some particular matters for which sufficiently extensive bibliographic references have already been published, the list of works cited at the end of the volume has been reduced so as to contain little more than 1400 items.

Even with these reductions, a significant increase in the size of the volume has been inevitable, and the necessity to contain this increase

[1] Actually the bibliography which I have collected and from which I have deduced these numbers is almost complete up to 1965, but is somewhat less so for the last two years.

within acceptable bounds has made it impossible for me to enter into too much detail in the most recent part of the theory. I have therefore adhered to the principle, already followed in the first edition, of reporting in detail only the results obtained for equations of the second order, and limiting myself, for equations of higher order and systems of equations, to a general indication of the methods used in the various works. Even for equations of the second order I could only in any case place emphasis on proofs of the most recent theorems.

Here in summary are the modifications made to the first edition: Chapter I, apart from some additions, has remained almost the same. Chapter II has been modified here and there in order to take into account the theorems from potential theory due to SOBOLEV and to CALDERON and ZYGMUND. In Chapter III, § 19 has been completely revised in order to connect the problem of the existence of fundamental solutions with the validity of the uniqueness theorem for CAUCHY'S problem and of the unique continuation property. In § 23 the results obtained in the study of the non-regular oblique derivative problems for equations in two variables have been discussed in rather more detail.

In Chapter IV, as well as various additions to §§ 26, 28, 29, 31, 32, the contents of §§ 27 and 30 are almost completely new, and, with changed titles, they are now devoted to local properties of the solutions of an elliptic equation and to the study of weak solutions of the boundary value problems, respectively.

In Chapter V, the title of which has been slightly changed, §§ 33, 34, 36 remain almost unaltered, § 35 has been supplemented with some new theorems, and §§ 37, 38, 39 have been completely revised. Of these three sections, the first two are devoted to the exposition of recent results on majorization, regularity, and existence in SOBOLEV spaces, while in § 39, which did not occur in the first edition, some results for the NEUMANN and oblique derivative problems are summarized. The contents of § 39 of the first edition have been divided among the preceding sections.

Finally, Chapters VI and VII are devoted, as in the first edition, to, first, nonlinear equations of second order, and second, to different problems from among the classical ones for equations of second order, equations of higher order, and systems of equations. Given the great quantity of results obtained in these areas in recent years, these chapters have had to be completely revised. In particular it has no longer been possible to put into Chapter VI a detailed exposition of the contributions due to CACCIOPPOLI, SCHAUDER, and LERAY to the theory of equations in two variables; however, these have been in large part absorbed by more recent results. Likewise in Chapter VII the section on degenerate elliptic equations has been placed at the end because its scope now includes equations of higher order.

Finally, I wish to thank Dr. Zane C. Motteler for the care and the dedication put into the work of translation and Springer Verlag for having taken this work into the Ergebnisse series in spite of its unusual dimensions.

Naples, 29 February 1968 CARLO MIRANDA

Translator's Preface

For many years I have been acquainted with the first edition of Professor Miranda's monograph, and I have found it a very useful reference book on several occasions. A couple of years ago it struck me that an English edition of this work would be quite useful, inasmuch as Italian is not well known among American mathematicians, primarily because French, German, and Russian are the usual languages required by graduate schools in this country. When I offered to translate this book Springer Verlag agreed that this should be done, and Professor Miranda agreed to produce an updated manuscript for this English second edition.

The translation was a difficult and demanding work for me, but at the same time it was a labor of love in my chosen field. Although many people have checked my manuscript for accuracy, I feel that I must be held solely responsible for any possible inaccuracies remaining in the text.

I wish to thank the Jesuit Research Council of Gonzaga University, which provided me with a grant which was very helpful in defraying the costs of typing and mailing the manuscript. Linda Balch did a fine job on the typing in spite of her many other duties. And most of all, Professor Miranda himself carefully inspected the manuscript, discovered many errors, and often supplied just the right turn of phrase to make a sentence read properly. I wish to thank Springer Verlag for the opportunity to work on this task with Professor Miranda, whose encyclopedic knowledge of the subject is sufficient to awe a younger mathematician.

Spokane, 4 July 1969 ZANE C. MOTTELER

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