

Thomas Mikosch

# Non-Life Insurance Mathematics

An Introduction with  
Stochastic Processes

 Springer

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