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Kenneth R. Meyer
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Introduction to Hamiltonian Dynamical Systems and the N-Body Problem

With 67 Illustrations

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Kenneth R. Meyer
Department of Mathematical Sciences
University of Cincinnati
Cincinnati, OH 45221-0025
USA

Glen R. Hall
Mathematics Department
Boston University
Boston, MA 02215
USA

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To the memory of Charles C. Conley

Preface

The theory of Hamiltonian systems is a vast subject which can be studied from many different viewpoints. This book develops the basic theory of Hamiltonian differential equations from a dynamical systems point of view. That is, the solutions of the differential equations are thought of as curves in a phase space and it is the geometry of these curves that is the important object of study. The analytic underpinnings of the subject are developed in detail. The last chapter on twist maps has a more geometric flavor. It was written by Glen R. Hall. The main example developed in the text is the classical N-body problem, i.e., the Hamiltonian system of differential equations which describe the motion of N point masses moving under the influence of their mutual gravitational attraction. Many of the general concepts are applied to this example. But this is not a book about the N-body problem for its own sake. The N-body problem is a subject in its own right which would require a sizable volume of its own. Very few of the special results which only apply to the N-body problem are given.

This book is intended for a first course at the graduate level. It assumes a basic knowledge of linear algebra, advanced calculus, and differential equations, but does not assume the advanced topics such as Lebesgue integration, Banach spaces, or Lie algebras. Some theorems which require long technical proofs are stated without proof, but only on rare occasions. The first draft of the book was written in conjunction with a course which was attended by engineering graduate students. The interests and background of these students influenced what was included and excluded.

This book was read by many individuals who made valuable suggestions and many corrections. The first draft was read and corrected by Ricardo Moena, Alan Segerman, Charles Walker, Zhangyong Wan, and Qui Dong Wang while they were students in a seminar on Hamiltonian systems. Gregg

Buck, Konstantin Mischaikow, and Dieter Schmidt made several suggestions for improvements to early versions of the manuscript. Dieter Schmidt wrote the section on the linearization of the equation of the restricted problem at the five libration points. Robin Vandivier found copious grammatical errors by carefully reading the whole manuscript. Robin deserves a special thanks. We hope that these readers absolve us of any responsibility.

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