# QUANTUM MECHANICS TWO VOLUMES BOUND AS ONE

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  - 32. Low energy Compton scattering. The Thomson formula.

APPENDIX C. VECTOR ADDITION COEFFICIENTS AND ROTATION MATRICES APPENDIX D. ELEMENTS OF GROUP THEORY GENERAL INDEX