Texts in Applied Mathematics 24

Springer
New York
Berlin
Heidelberg
Barcelona
Budapest
Hong Kong
London
Milan
Paris
Santa Clara
Singapore
Tokyo

Texts in Applied Mathematics

1. *Sirovich:* Introduction to Applied Mathematics.
2. *Wiggins:* Introduction to Applied Nonlinear Dynamical Systems and Chaos.
3. *Hale/Koçak:* Dynamics and Bifurcations.
4. *Chorin/Marsden:* A Mathematical Introduction to Fluid Mechanics, 3rd ed.
5. *Hubbard/West:* Differential Equations: A Dynamical Systems Approach: Ordinary Differential Equations.
6. *Sontag:* Mathematical Control Theory: Deterministic Finite Dimensional Systems.
7. *Perko:* Differential Equations and Dynamical Systems, 2nd ed.
8. *Seaborn:* Hypergeometric Functions and Their Applications.
9. *Pipkin:* A Course on Integral Equations.
10. *Hoppensteadt/Peskin:* Mathematics in Medicine and the Life Sciences.
11. *Braun:* Differential Equations and Their Applications, 4th ed.
12. *Stoer/Bulirsch:* Introduction to Numerical Analysis, 2nd ed.
13. *Renardy/Rogers:* A First Graduate Course in Partial Differential Equations.
14. *Banks:* Growth and Diffusion Phenomena: Mathematical Frameworks and Applications.
15. *Brenner/Scott:* The Mathematical Theory of Finite Element Methods.
16. *Van de Velde:* Concurrent Scientific Computing.
17. *Marsden/Ratiu:* Introduction to Mechanics and Symmetry.
18. *Hubbard/West:* Differential Equations: A Dynamical Systems Approach: Higher-Dimensional Systems.
19. *Kaplan/Glass:* Understanding Nonlinear Dynamics.
20. *Holmes:* Introduction to Perturbation Methods.
21. *Curtain/Zwart:* An Introduction to Infinite-Dimensional Linear Systems Theory.
22. *Thomas:* Numerical Partial Differential Equations: Finite Difference Methods.
23. *Taylor:* Partial Differential Equations: Basic Theory.
24. *Merkin:* Introduction to the Theory of Stability.

David R. Merkin

Introduction to the Theory of Stability

Translated and Edited by Fred F. Afagh and Andrei L. Smirnov

With 94 Illustrations

Springer

David R. Merkin
St. Petersburg State University
(Retired)

Translators:
Andrei L. Smirnov
Faculty of Mathematics and Mechanics
St. Petersburg State University
St. Petersburg, Russia

Fred F. Afagh
Department of Mechanical and Aerospace Engineering
Carleton University
Ottawa, ON K1S 5B6, Canada

Mathematics Subject Classifications: 73hxx, 58f10, 70j25, 70k20, 93dxx

Merkin, David Rakhmil'evich.
[Vvedenie v teoriiu ustoichivosti dvizheniia. English]
Introduction to the theory of stability / David R. Merkin ; translated from the Russian by Andrei L. Smirnov, Fred Afagh.
p. cm. — (Texts in applied mathematics ; 24)
Includes bibliographical references and index.

ISBN-13: 978-1-4612-8477-2 e-ISBN-13: 978-1-4612-4046-4

DOI: 10.1007/978-1-4612-4046-4

1. Stability. 2. Motion. I. Title. II. Series.
QA871.M4713 1996
531.3—DC20 96-15277

Printed on acid-free paper.

Softcover reprint of the hardcover 1st edition 1997

Production managed by Terry Kornak; manufacturing supervised by Joe Quatela.
Typeset from the author's LaTeX file using Springer's svsing.sty macro.

9 8 7 6 5 4 3 2 1

Preface to the English Edition

Many books on stability theory of motion have been published in various languages, including English. Most of these are comprehensive monographs, with each one devoted to a separate complicated issue of the theory. Generally, the examples included in such books are very interesting from the point of view of mathematics, without necessarily having much practical value. Usually, they are written using complicated mathematical language, so that except in rare cases, their content becomes incomprehensible to engineers, researchers, students, and sometimes even to professors at technical universities.

The present book deals only with those issues of stability of motion that most often are encountered in the solution of scientific and technical problems. This allows the author to explain the theory in a simple but rigorous manner without going into minute details that would be of interest only to specialists. Also, using appropriate examples, he demonstrates the process of investigating the stability of motion from the formulation of a problem and obtaining the differential equations of perturbed motion to complete analysis and recommendations. About one fourth of the examples are from various areas of science and technology. Moreover, some of the examples and the problems have an independent value in that they could be applicable to the design of various mechanisms and devices.

The present translation is based on the third Russian edition of 1987. The author has complemented this translation by inserting some brief additional explanations, by including an appropriate list of references from publications in the United States and the United Kingdom and by adding proper exercises which in most cases are provided either with answers or with hints on how to solve them. The author hopes that this book will be useful for English-language readers.

Professors Andrei L. Smirnov (St. Petersburg University, Russia) and Fred F.

Afagh (Carleton University, Canada) coordinated and managed all the necessary work to translate and edit the book. The author would like to express his most heartfelt gratitude to the translators and editors of this book.

David R. Merkin

Preface to the Third Russian Edition

The present book is one of the textbooks published by the "Nauka" Publishing House as an addition to *A Course In Theoretical Mechanics* by N.V. Butenin, Ya.L. Lunc, and D.R. Merkin. The reason for publishing these textbooks is that students of technical universities need to become more closely acquainted with a number of more important topics than those dealt with in that introductory course. The textbooks included in the proposed series are devoted to such topics as analytical mechanics, stability theory of motion, theory of oscillations, theory of gyroscopes, and impact theory. This list is to be continued in the future.

Numerous books have been published in the Soviet Union [10, 11, 13, 23, 27, 29, 56, 59, 67, 72, 130], in all of which the stability theory of motion is presented at various levels of completeness and from different points of view. Some of these are scientific monographs rather than textbooks. Generally, such monographs are intended for students from faculties of mathematics and theoretical mechanics with an intensive background in mathematics. Otherwise, these monographs are too complicated for engineering students at technical universities.

Basically, the present book is written for students at technical universities as well as for engineers and scientists who use the theory of motion stability in their work. In this regard, the mathematics used in the book does not exceed the level of knowledge taught in most engineering faculties. Any required advanced mathematics is included in the book.

In order to simplify the book, we initially consider autonomous systems. The stability of motion of nonautonomous systems is presented only in Chapter 7. Therefore, some theorems are proved under explicitly stated simplified conditions, with references to where the proof of these theorems under general conditions may be found.

The most effective method of studying stability of motion, i.e., the direct method of Liapunov and the stability in the first approximation, is the main focus of this book. Some chapters deal with this topic based upon the type of applied forces. Also, the stability of nonautonomous systems, including those in which the perturbed motion is described by linear differential equations with periodical coefficients, is presented.

The application of the direct method of Liapunov to the stability analysis of automatic control systems is considered in Chapter 8, and finally, Chapter 9 is devoted to frequency methods of analyzing stability of motion.

In recognition of the fact that those who are introduced to the stability theory of motion for the first time usually experience a great deal of difficulty in applying the theory to the solution of practical problems, much attention is allotted to the selection and solution of examples from various disciplines of science and technology. A sizable number of the examples and problems stand by themselves as significant exercises.

This book has grown out of several years of lectures by the author at the Faculty of Postgraduate Studies at Leningrad (St. Petersburg) State University. Numerous consultations with engineers and scientists at scientific research institutes in Leningrad on different aspects of the theory of motion stability and its applications have influenced the nature and content of this book.

New examples are included in the third edition (the first and second editions were published in 1971 and 1976, respectively). Some recent articles published since 1976 are included in this edition. Also, in revising the text of the book some misprints have been corrected.

The significant contributions and suggestions of Correspondent Members of the USSR Academy of Science A.I. Lurie, and V.V. Rumyantsev, as well as those of Associate Professors B.A. Smolnikov and B.L. Mintsberg in preparation of the first edition of this book are gratefully acknowledged. A major part of Chapter 9 was prepared by A.H. Gelig. Many valuable suggestions on the second and third editions were made by Correspondent Member of the USSR Academy of Science V.V. Rumyantsev. The author would like to express his most heartfelt gratitude to all of these people.

David R. Merkin

From the Editors

Several features make the book by Prof. D.R. Merkin unique among many books on the theory of stability of motion that have been published in various languages.

The main advantage of the book is its simple yet simultaneously rigorous presentation of the concepts of the theory, which often are presented in the context of applied problems with detailed examples demonstrating effective methods of solving practical problems.

All the classical theories of Lagrange, Liapunov, Chetaev, Krasovsky, Thomson and Tait, Hurwitz, Nyquist and others as well as new results obtained by the author are presented in this text. These new results deal with investigating the stability of motion under gyroscopic, dissipative, and nonconservative position forces (Section 6.7 and 6.8). Also presented are sufficient conditions for asymptotic stability of a system with nonlinear rigidity and damping that are explicit functions of time (Section 7.4).

Examples constitute about 25% of the entire volume and cover various areas in science and engineering. Moreover, some of the examples possess an independent value in that they could be used in the analysis of various real structures and mechanisms.

The above features have made the *Introduction to the Theory of Stability of Motion* the most popular textbook in its field at faculties of matematics and mechanics as well as engineering faculties in Russian universities. The present translation is based on the third Russian edition of 1987.

The present book is a result of the scientific cooperation of the Departments of Theoretical and Applied Mechanics of the Faculty of Mathematics and Mechanics at St. Petersburg State University in Russia and the Department of Mechanical and Aerospace Engineering at Carleton University in Ottawa, Canada. The author and

the editors would like to express their special thanks to Prof. John A. Goldak and Prof. D.R.F. Taylor of Carleton University, whose initial support of the program of scientific cooperation between St. Petersburg and Carleton University made it possible to prepare this manuscript.

This work was supported in part by the Russian Foundation for Fundamental Research and the Soros International Foundation under grant # 5000.

The editors translated and edited the book and did the typesetting using LaTeX. We would like to thank our colleagues and students Mrs. Yu. Mochalova, Ms. O. Bukashkina, Mrs. V. Sergeeva, Mr. V. Piotrovich, Mr. A. Mironov, and Mr. I. Malygin for their help in translating and typesetting the manuscript. We also would like to thank Mr. N. Filippov, Mr. S. Chernov, and Mr. S. Zakharov for their excellent drawings.

Fred F. Afagh and Andrei L. Smirnov

About the Author and the Editors

David R. Merkin is a leading Russian specialist in the theory of motion stability. He continues the tradition of Russian (Soviet) research in this area that is associated with such names as Liapunov, Chetaev, Chaplygin, and Krasovsky. Prof. D. Merkin was born in 1912. In 1940 he graduated (with honors) from the Faculty of Mathematics and Mechanics at Leningrad State University. From 1941 he was at the front during the Second World War until 1945, when he was demobilized with the rank of major. In 1947 he received his degree as a Candidate of Sciences (Ph.D.). In 1949 he obtained new results in stability of gyroscopic systems and investigated the general properties of such systems at high rotational speeds of rotors by developing the methods proposed by Thomson (Lord Kelvin) and Tait. He obtained the degree of Doctor of Sciences in 1957, following which he served for almost twenty-five years as the chair of the Department of Theoretical Mechanics at Leningrad Water Transportation Institute Institute. His course entitled "Stability of Motion" was delivered for more than 25 years at the Faculty of Postgraduate Studies at Leningrad State University. During those years, he published six books, including Gyroscopic Systems (1956, 2nd ed. 1974) and Introduction to the Theory of Stability of Motion (1971, 2nd ed. 1976, 3rd ed. 1987) and more than fifty papers. Since 1953, Professor Merkin has served as a member of the Scientific Council of General Mechanics of the Russian Academy of Sciences as well as a member of the National Committee of Theoretical and Applied Mechanics of this Academy since 1966.

Fred Farhad Afagh is an Associate Professor and Associate Chair (Graduate Studies) in the Department of Mechanical and Aerospace Engineering at Carleton University, Ottawa, Canada. He obtained his Bachelor of Science in Civil

Engineering as well as his Master of Science in Structural Engineering from Washington State University, in 1969 and 1971, respectively. After gaining more than ten years of industrial experience as a consulting engineer in North America and overseas, in 1986 he obtained his Ph.D. in Engineering Mechanics from the University of Waterloo, Canada, where he studied under the Late Professor H.H.E. Leipholz. Dr. Afagh was the technical editor of the Proceedings of the Second International Symposium on Structural Control, Ed. H.H.E. Leipholz. His papers on dynamic analysis and stability of linearly elastic and non-selfadjoint systems with distributed masses as well as stress analysis in isotropic and anisotropic elasticity using the boundary integral equation method have been published in *The Journal of Sound and Vibration*, *Archive of Applied Mechanics*, *International Journal of Solids and Structures*, and other journals.

Andrei L. Smirnov was born in Leningrad, USSR, in 1956. He is an Associate Professor in the Department of Theoretical and Applied Mechanics of the Faculty of Mathematics and Mechanics at St. Petersburg State University (formerly Leningrad State University). He graduated in Applied Mathematics from Leningrad State University in 1978 and obtained his Ph.D. in Mechanics of Solids from the same university in 1981 with the dissertation *Vibrations of Rotating Shells of Revolution*. Dr. Smirnov is the editor as well as one of the authors of two books: *Asymptotic Methods in Mechanics* and *Asymptotic Methods in Mechanics of Thin Structures*. His papers on asymptotic and numerical methods in mechanics of thin structures have been published in *Transactions of the ASME*, *Transactions of the CSME*, *Technische Mechanik*, *Vestnik Leningradskogo Universiteta*, and other journals.

Contents

List of Figures