MALCOLM ADAMS VICTOR GUILLEMIN

# Measure Theory and Probability

Birkhäuser Boston • Basel • Berlin Malcolm Adams Department of Mathematics University of Georgia Athens, Georgia 30602 Victor Guillemin Department of Mathematics MIT Cambridge, MA 02139

#### Library of Congress Cataloging-in-Publication Data

Adams, Malcolm Ritchie.
Measure theory and probability / Malcom Adams, Victor Guillemin.
p. cm.
Originally published: Monterey Calif. : Wadsworth & Brooks/Cole
Advanced Books and Software, c 1986.
Includes bibliographical references and index.

alk. paper) 1. Measure theory. 2. Probabilities. I. Guillemin, V., 1937-. II. Title. [QA273.A414 1995] 515'.42--dc20 95-46511

Printed on acid-free paper © 1996 Birkhäuser Boston Reprinted with corrections from the 1986 Wadsworth edition.

Birkhäuser

CIP

Copyright is not claimed for works of U.S. Government employees. All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without prior permission of the copyright owner.

Permission to photocopy for internal or personal use of specific clients is granted by Birkhäuser Boston for libraries and other users registered with the Copyright Clearance Center (CCC), provided that the base fee of \$6.00 per copy, plus \$0.20 per page is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923, U.S.A. Special requests should be addressed directly to Birkhäuser Boston, 675 Massachusetts Avenue, Cambridge, MA 02139, U.S.A.

ISBN 978-1-4612-6899-4 DOI 10.1007/978-1-4612-0779-5 ISBN 978-1-4612-0779-5 (eBook)

Typesetting by ASCO Trade Typesetting, Hong Kong

98765432

# Contents

## Chapter 1 Measure Theory 1

- §1.1 Introduction 1
- §1.2 Randomness 14
- \$1.3 Measure Theory 24
- §1.4 Measure Theoretic Modeling 42

# Chapter 2 Integration 53

- §2.1 Measurable Functions 53
- §2.2 The Lebesgue Integral 60
- §2.3 Further Properties of the Integral; Convergence Theorems 72
- §2.4 Lebesgue Integration versus Riemann Integration 82
- §2.5 Fubini Theorem 89
- §2.6 Random Variables, Expectation Values, and Independence 102
- §2.7 The Law of Large Numbers 110
- §2.8 The Discrete Dirichlet Problem 115

### Chapter 3

### Fourier Analysis 118

- \$3.1  $\pounds^1$ -Theory 118
- \$3.2  $\pounds^2$ -Theory 124
- §3.3 The Geometry of Hilbert Space 130
- §3.4 Fourier Series 137
- §3.5 The Fourier Integral 145
- \$3.6 Some Applications of Fourier Series to Probability Theory 156
- §3.7 An Application of Probability Theory to Fourier Series 164
- §3.8 The Central Limit Theorem 170

Contents

Appendix AMetric Spaces178Appendix BOn  $\mathcal{L}^p$  Matters183Appendix CA Non-Measurable Subsetof the Interval (0, 1]199References202Index203