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Comprehensive Mathematics for Computer Scientists 1

Sets and Numbers, Graphs and Algebra,
Logic and Machines, Linear Geometry

With 82 Figures



Springer

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