

# Contents

Preface	ix
Preface to the English Translation	xi
Objectives	xiii
Chapter 1. Morse Theory on Surfaces	1
1.1. Critical points of functions	1
1.2. Hessian	3
1.3. The Morse lemma	8
1.4. Morse functions on surfaces	14
1.5. Handle decomposition	22
a. The case when the index of $p_0$ is zero	26
b. The case when the index of $p_0$ is one	26
c. The case when the index of $p_0$ is two	29
d. Handle decompositions	30
Summary	31
Exercises	31
Chapter 2. Extension to General Dimensions	33
2.1. Manifolds of dimension $m$	33
a. Functions on a manifold and maps between manifolds	33
b. Manifolds with boundary	34
c. Functions and maps on manifolds with boundary	38
2.2. Morse functions	41
a. Morse functions on $m$ -manifolds	41
b. The Morse lemma for dimension $m$	44
c. Existence of Morse functions	47
2.3. Gradient-like vector fields	56
a. Tangent vectors	56

b. Vector fields	61
c. Gradient-like vector fields	63
2.4. Raising and lowering critical points	69
Summary	71
Exercises	71
Chapter 3. Handlebodies	73
3.1. Handle decompositions of manifolds	73
3.2. Examples	83
3.3. Sliding handles	105
3.4. Canceling handles	120
Summary	130
Exercises	131
Chapter 4. Homology of Manifolds	133
4.1. Homology groups	133
4.2. Morse inequality	141
a. Handlebodies and cell complexes	141
b. Proof of the Morse inequality	147
c. Homology groups of complex projective space $CP^m$	147
4.3. Poincaré duality	148
a. Cohomology groups	148
b. Proof of Poincaré duality	150
4.4. Intersection forms	158
a. Intersection numbers of submanifolds	159
b. Intersection forms	159
c. Intersection numbers of submanifolds and intersection forms	153
Summary	164
Exercises	164
Chapter 5. Low-dimensional Manifolds	167
5.1. Fundamental groups	167
5.2. Closed surfaces and 3-dimensional manifolds	173
a. Closed surfaces	173
b. 3-dimensional manifolds	181
5.3. 4-dimensional manifolds	186
a. Heegaard diagrams for 4-dimensional manifolds	186
b. The case $N = D^4$	190
c. Kirby calculus	194

## CONTENTS

vii

Summary	197
Exercises	197
A View from Current Mathematics	199
Answers to Exercises	203
Bibliography	213
Recommended Reading	215
Index	217