

Martzloff · A History of *Chinese Mathematics*

Springer-Verlag
Berlin Heidelberg GmbH



LI SHANLAN
1811-1882

From the July 1877 issue of the *Ghezi huibian*
(The Chinese scientific and industrial magazine)

Jean-Claude Martzloff

A HISTORY OF
Chinese Mathematics

With Forewords by
Jaques Gernet and Jean Dhombres

With 185 Figures



Springer

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Title of the French original edition:
Histoire des mathématiques chinoises. © Masson, Paris 1987

Cover Figure: After an engraving taken from the *Zhiming suanfa* (Clearly explained computational [arithmetical] methods). This popular book, edited by a certain Wang Ren'an at the end of the Qing dynasty, is widely influenced by Cheng Dawei's famous *Suanfa tongzong* (General source of computational methods) (1592). Cf. Kodama Akihito (2'), 1970, pp. 46–52.

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Library of Congress Cataloging-in-Publication Data

Martzloff, Jean-Claude.

[*Histoire des mathématiques chinoises.* English]

A history of Chinese mathematics / Jean-Claude Martzloff ; with a foreword by J. Gernet and J. Dhombres ; [translator, Stephen S. Wilson].

p. cm.

Includes bibliographical references and indexes.

ISBN 978-3-540-33782-9

ISBN 978-3-540-33783-6 (eBook)

DOI 10.1007/978-3-540-33783-6

1. Mathematics--China--History. I. Title.

QA27.C6M3613 1997

510'.951--dc20

95-25671

CIP

ISBN 978-3-540-33782-9

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Originally published by Springer-Verlag 1997

Softcover reprint of the hardcover 1st edition 1997

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Cover design: Erich Kirchner, Heidelberg

Typesetting: Editing and reformatting of the translator's input files using a Springer T_EX macro package

SPIN 10028408

41/3143 - 5 4 3 2 1 0 - Printed on acid-free paper

To France Alice

Foreword by J. Gernet

The uses of numbers, their links with the socio-political system, their symbolic values and their relationship to representations of the universe say a great deal about the main characteristics of a civilisation. Although our mathematics has now become the common heritage of humanity, our understanding of mathematics is essentially based on a tradition peculiar to ourselves which dates back to ancient Greece; in other words, it is not universal. Thus, before we can begin to understand Chinese mathematics, we must not only set aside our usual ways of thinking, but also look beyond mathematics itself. At first sight, Chinese mathematics might be thought of as empirical and utilitarian since it contains nothing with which we are familiar; more often than not it contains no definitions, axioms, theorems or proofs. This explains, on the one hand, earlier unfavourable judgements of Chinese mathematics and, on the other hand, the amazement generated by the most remarkable of its results. The Chinese have always preferred to make themselves understood without having to spell things out. "I will not teach anyone who is not enthusiastic about studying," said Confucius. "I will not help anyone who does not make an effort to express himself. If, when I show someone a corner, that person does not reply with the three others, then I will not teach him." However, the Chinese have indulged their taste for conciseness and allusion, which is so in keeping with the spirit of their language, to the extent that they detest the heaviness of formal reasoning. This is not a case of innate incapacity, since their reasoning is as good as ours, but a fundamental characteristic of a civilisation. Moreover, this loathing of discourse is accompanied by a predilection for the concrete. This is clearly shown by their methods of teaching mathematics, in which the general case is illustrated by operational models the possibilities of extension of which they record directly, via comparisons, parallels, manipulation of numbers, cut-out images, and reconstruction and rotation of figures. As J.-C. Martzloff notes, for the Chinese, numbers and figures relate to objects rather than to abstract essences. This is the complete antithesis to the Greeks, who rejected everything that might evoke sensory experience, and runs counter to the Platonic concept of mathematics as the theoretical science of numbers, an objective science concerned with the abstract notions of units and magnitudes "which enable the soul to pass from the ever-changing world to that of truth and essence." For the Chinese, on the other hand, numbers formed a component part of the changing world to which they adapt; for instance, there was no distinction between

counting-rods and the divinatory rods which were used to create hexagrams from combinations of the *yin* and *yang* signs. Chinese diviners are credited with astonishing abilities as calculators. In China, there was a particularly close link between mathematics and the portrayal of cosmology and, as Marcel Granet wrote, numbers were used “to define and illustrate the organisation of the universe.” This may explain the importance of directed diagrams and the fundamental role of position in Chinese algebra (which determines the powers of ten and the powers of the unknown on counting surfaces). The number 3 is sometimes used as an approximate equivalent of the number π because it is the number of the Heavens and the circle, in the same way that 2 is the number of the square and of the Earth. The set-square and compass are the attributes of Fuxi and Nüwa, the mythical founders of the Chinese civilisation and the persistence in Chinese mathematics of a figure such as the circle inscribed in a right-angled triangle (right-angled triangles form the basis for a large number of algebraic problems) cannot be simply put down to chance. Chinese mathematics was oriented towards cosmological speculations and the practical study of the hidden principles of the universe as much as towards questions with a practical utility. It can scarcely be distinguished from an original philosophy which placed the accent on the unity of opposites, relativity and change.

Martzloff provides not only an excellent analysis of the remaining testimonies to the long history of Chinese mathematics (many works have disappeared and many procedures which were only passed on by example and practice have vanished without trace) but a study of all aspects of its history, which covers contacts and borrowings, the social situation of mathematicians, the place of mathematics in the civilisation and Western works translated into Chinese from the beginning of the 17th century, including the problems involved in the translation of these works. There emerge an evolution with its apogee in the 12th and 13th centuries and a renaissance stimulated by the contribution of Western mathematics in the 17th and 18th centuries. This admirably documented book, in which the author has made every attempt not to “dress Chinese mathematics in clothes which it never wore,” will be an indispensable work of reference for a long time to come.

Jacques GERNET

Honorary Professor at the Collège de France

Foreword by J. Dhombres

Since the encyclopedist movement of the 18th century which was in harmony with the ideas of the Enlightenment, we have got so used to viewing science as a common human heritage, unlike an individual sense of citizenship or a specific religion, that we would like to believe that its outward forms are universal and part of a whole, which, if it is not homogeneous is at least compulsory and unbounded. Thus, in a paradoxical return to ethnocentrism, it seems quite natural to us that, even though it means taking liberties with the writing of history, this science was that described by Aristotle's logical canons, Galileo's mathematical techniques and Claude Bernard's rational experimentalism.

Moreover, we have also assumed that, as far as mathematics is concerned, there is only one model, the evolution of which was essentially fixed from the origins of a written civilisation by the immutable order of the rules of the game, namely axioms, theorems and proofs displayed in a majestic architectural sequence in which each period added its name to the general scheme by contributing a column, an architrave, a marble statue or a more modest cement. One name, that of Euclid, whose *Elements* were used as a touchstone to test whether a work was worthy of being called "mathematical," has resounded from generation to generation since the third century BC. The model transcended mathematical specialisation (still suspected of favouring useless mysteries) since so many thinkers laboured to present their ideas *more geometrico*. They would have been insulted by the suggestion that they should replace this expression by another, such as "as prescribed by the School of Alexandria," which emphasized the geographical attachment. These thinkers believed that they proceeded in accordance with the universal rules of the human mind.

The civilisations of the Mediterranean Basin and, later, those of the Atlantic were not wrong to venerate the axiomatic method. They also knew how to yield graciously to mathematical approaches, such as the discovery of differential and integral calculus at the end of the 17th century, which were initially rightly judged to be less rigorous. Thus, apart taking an interest in another culture, and another way of thinking, not the least merit of a history of mathematics outside the influence of Euclid and his accomplices would be to improve our grasp of the strength and penetration of the Euclidean approach. To put it more prosaically, without risk of contradiction by French and Chinese gourmets, *doufu* and *haishen* taste better once one has tried *foie gras* and oysters!

Fortunately, there exist different types of mathematics, such as those which have developed continuously and fruitfully over approximately sixteen centuries in the basins of the Yellow and Blue Rivers. Should we still refuse these the right to the 'mathematical' label because there are as yet very few well-documented books about them? Certainly not, since we now have the present *Histoire des Mathématiques Chinoises* from the expert pen of Jean-Claude Martzloff. This enthusiastically describes the one thousand and one linguistic and intellectual pitfalls of the meeting between the end of the Ming culture and that of the Qing successors. This meeting involved Euclid or, rather, a certain Euclid resulting from the Latin version of the *Elements* generated in 1574 by Clavius. In fact, Clavius was the master of the Jesuit Ricci (otherwise known in Peking under the name of Li Madou) who translated the first six books of the mathematician from Alexandria into Chinese at the beginning of the 17th century.

Unfortunately, although the Jesuits placed the translation of mathematics before that of the Holy Scriptures, they did not have access to original Chinese mathematics such as the algebraic and computational works of the brilliant Chinese foursome of the 13th century Yang Hui, Li Zhi, Qin Jiushao and Zhu Shijie. What would they have made of this, when their own mathematical culture was so rich?

For it is a most surprising historical paradox that this meeting between the West and China took place at a time when a complete scientific reversal was under way in the West (the change occurred over a few short years). Sacrobosco's astronomy of the planets, a direct descendant of that of Ptolemy, which the 'good fathers' took with them on their long sea journey to distant Cathay, even when adapted in response to scholarly lessons received at the College of Rome where the Gregorian calendar was reformed in 1572, was very different from that given by Kepler in his *Astronomia Nova* in 1609. The theoretical and intangible reflections of the 14th-century mechanistic schools of Paris and Oxford were suddenly realised in the true sense, when they were applied in the real world by Galileo when he established the law of falling bodies. The West was seen to be on the outside in well-worn clothes, although the Far-East had forgotten its mathematical past. However, it is true that the *Suanfa tongzong* (General source of computational methods) which was issued in 1592, would not have disgraced a 16th-century collection of Western arithmetics! However, on both the Chinese and the Western sides, originality was to be found elsewhere.

It is because we are well aware of the originality of Galileo and Descartes that our interest turns to the above four Chinese 13th-century mathematicians. Their originality is so compelling that we are overcome with a desire to know more about how they thought and lived, the sum total of their results and the links between their works and their culture. In short, our curiosity is excited, and the merit of this book is that it leads through both the main characters and the main works.

But what is the intended audience of this book on the history of mathematics, given that its unique nature will guarantee its future success and longevity through the accumulation of specialised scholarly notices and, above

all, more broadly, through reflexions by specialists in all areas? Is this book solely for austere scholars who use numerical writings to measure exchanges between the Indus and the Wei and between the Arabic-speaking civilisation and the Tang? Is it solely intended for those interested in the origin of the zero or the history of decimal positional numeration?

How narrow the specialisations of our age are, that it is necessary to tell ill-informed readers as much about the affairs and people of China as about modern mathematics, to enable them to find spiritual sustenance in the pages of this book. May they not be frightened by the figures or by a few columns of symbols. May they be attracted by the Chinese characters, as well as by the arrangements of counting-rods, since these determine a different policy in graphical space. Where can a mathematician or historian of China find so much information or a similar well-organised survey of sources? Where can one find such a variety of themes, ranging from the interpretation of the mathematical texts themselves to a description of the role of mathematics in this civilisation, which was strained by literary examinations from the Tang, preoccupied with the harmony between natural kingdoms, and partial to numerical emblems (as Marcel Granet breathtakingly shows in his *La Pensée Chinoise*)?

I shall only comment on a number of questions about this Chinese mathematics and a number of very general enigmas which have nothing to do with this exotic and quaint *enigma cinese*.

Firstly, there is the question of a difference in status between the mathematics of practitioners and that of textbooks intended for teaching purposes. Broadly speaking, as far as China is concerned, it is mainly textbooks which have come down to us, worse still, these are textbooks which belong to an educational framework which placed great value on the oral tradition and on the memorising of parallel, rhyming formulae. How could we describe 18th-century French mathematics if we only had access to the manuals due to Bézout, Clairaut or Bougainville? Moreover, should not textbooks be written in such a way that they adhere to the practice of mathematical research of a period, as Monge, Lagrange and Laplace deigned to believe during the French Revolution? Should greater importance be placed on metonymy, the passage from the particular to the general, based on the a priori idea that local success should reveal a hidden structure, even during the training procedure? Is a vague sense of analogy a sufficient basis on which to found an education at several successive theoretical levels? Thus, the history of the mathematics developed in Hangzhou, or any other capital, gives the teacher something to think about.

I have already mentioned the importance of the encounter between the West and the East in the 17th century, with which the French reader is familiar through such important works as J. Gernet's *Chine et Christianisme*, and J. D. Spence's *The Memory Palace of Matteo Ricci*. Unfortunately, these texts pass hurriedly over important scientific aspects. Thus, J.-C. Martzloff has provided an original contribution to an ongoing interrogation.

Finally, there is the question of whether or not the commentary plays a major role in the Chinese mathematical tradition. There is always a tendency

to consider mathematicians as a taciturn breed; the very existence of a commentary on a mathematical text may come as a surprise. Arabic-speaking mathematicians distinguished between commentaries (tafsīr), “redactions” (or taḥrīr) and revisions (or iṣlah). They may have done this because they were confronted with the Euclidean tradition which they transmitted and supplemented. The fact is that, within the framework of a theory, the axiomatic approach only ceases once the individual role of each axiom, the need for that axiom and its relative importance amongst the legion of other axioms have been determined. However, the Chinese, impervious to axiomatic concerns, added their own commentaries. At the beginning of the third century AD, Liu Hui, in his commentary on the Computational Prescriptions in Nine Chapters, gave one of the rare proofs of the Chinese mathematical corpus. Can one thus continue to believe that mathematical texts were treated like the Classics, with all the doxology accumulated over the generations, like a true Talmud in perpetual motion? Did mathematics feed so heartily on the sap secreted by a period, a culture or an anthropology that a commentary was necessary? The numerical examples chosen by mathematicians to construct the gates at the four cardinal points of a Chinese town, and the calculation of the tax base constitute a precise revelation of a lost world and are useful in archaeology. But beyond this, does not the mathematics developed by a generation reveal its innermost skeletal structure, much like an X-ray?

What a lot of questions now arise about this area of the history of Chinese mathematics, which at first seemed so compartmentalised, so technical, and scarcely worthy of the general interest of historians or, even less, the interest of those who study the evolution of mental attitudes. After studying general aspects of Chinese mathematics in the first part of his book, J.-C. Martzloff strikes an admirable balance by encouraging us to delve into the second part which concerns the authors and their works. In short, it is difficult not to be fond of his survey, which is solidly supported by bibliographic notes.

It is to be hoped that this first French edition will give rise to publications of the original Chinese texts (with translations) so that we would have a corpus of Chinese mathematics, in the same way that we are able to consult the Egyptian corpus, the Greek corpus and, to a lesser extent, the Babylonian corpus.

Jean DHOMBRES

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人皆知有用之用而莫知无用之用也

*Everyone knows the usefulness of the useful,
but no one knows the usefulness of the useless.*

Zhuangzi (a work attributed to
ZHUANG ZHOU (commonly known as ZHUANGZI)),
ch. 4, "The world of men"

Preface

Since the end of the 19th century, a number of specialised journals, albeit with a large audience, have regularly included articles on the history of Chinese mathematics, while a number of books on the history of mathematics include a chapter on the subject. Thus, the progressive increase in our knowledge of the content of Chinese mathematics has been accompanied by the realisation that, as far as results are concerned, there are numerous similarities between Chinese mathematics and other ancient and medieval mathematics. For example, Pythagoras' theorem, the double-false-position rules, Hero's formulae, and Ruffini-Horner's method are found almost everywhere.

As far as the reasoning used to obtain these results is concerned, the fact that it is difficult to find rational justifications in the original texts has led to the *reconstitution* of proofs using appropriate tools of present-day elementary algebra. Consequently, the conclusion that Chinese mathematics is of a fundamentally algebraic nature has been ventured.

However, in recent decades, new studies, particularly in China and Japan, have adopted a different approach to the original texts, in that they have considered the Chinese modes of reasoning, as these can be deduced from the rare texts which contain justifications. By studying the results and the methods explicitly mentioned in these texts hand in hand, this Chinese and Japanese research has attempted to reconstruct the conceptions of ancient authors within a given culture and period, without necessarily involving the convenient, but often distorting, social and conceptual framework of present-day mathematics.

This has led to a reappraisal of the relative importance of different Chinese sources; texts which until recently had been viewed as secondary have now become fundamental, by virtue of the wealth of their proofs. However, most of all, this approach has brought to the fore the key role of certain operational procedures which form the backbone of Chinese mathematics, including heuristic computational and graphical manipulations, frequent recourse to geometrical dissections and instrumental tabular techniques in which the position of physical objects representing numbers is essential. Thus, it has become increasingly clear that within Chinese mathematics, the contrasts between algebra and geometry and between arithmetic and algebra do not play the same role as those in mathematics influenced by the axiomatico-deductive component of the Greek tradition. It is now easier to pick out the close bonds between apparently unrelated Chinese computational techniques

(structural analogy between the operation of arithmetical division and the search for the roots of polynomial equations, between calculations on ordinary fractions and rational fractions, between the calculation of certain volumes and the summation of certain series, etc.). It is in this area that the full richness of studies which focus on the historical context without attempting to clothe Chinese mathematics in garments which it never wore becomes apparent.

Beyond the purely technical aspect of the history of mathematics, the attention given to the context, suggests, more broadly, that the question of other aspects of this history which may provide for a better understanding of it is being addressed. In particular, we point to:

- The question of defining the notion of mathematics from a Chinese point of view: was it an art of logical reasoning or a computational art? Was it arithmetical and logistical or was it concerned with the theory of numbers? Was it concerned with surveying or geometry? Was it about mathematics or the *history* of mathematics?
- The important problem of the destination of the texts. Certain texts may be viewed as accounts of research work, others as textbooks, and others still as memoranda or formularies. If care was not taken to distinguish between these categories of texts, there would be a danger of describing Chinese mathematical thought solely in terms of ‘Chinese didactic thought’ or ‘Chinese mnemonic thought.’ The fact that a textbook does not contain any proofs does not imply that its author did not know how to reason; similarly, the fact that certain texts contain summary proofs does not imply that the idea of a well-constructed proof did not exist in China: one must bear in mind, in particular, the comparative importance of the oral and written traditions in China.
- The question of the integration into the Chinese mathematical culture of elements external to it. The history of Chinese reactions to the introduction of Euclid’s *Elements* into China in the early 17th century highlights, in particular, the differences between systems of thought.

It is with these questions in mind that we have divided this book into two parts, the first of which is devoted to the context of Chinese mathematics and the second to its content, the former being intended to clarify the latter. We have not attempted to produce an encyclopedic history of Chinese mathematics, but rather to analyse the general historical context, to test results taken for granted against the facts and the original texts and to note any uncertainties due to the pooriness of the sources or to the limitations of current knowledge about the ancient and medieval world.

Acknowledgements

I should firstly like to express my gratitude to Jacques Gernet, Honorary Professor at the Collège de France (Chair of Social and Intellectual History of China), Jean Dhombres, Directeur d'Etudes at the Ecole des Hautes Etudes en Sciences Sociales and Director of the U.P.R. 21 (C.N.R.S., Paris) for their constant support throughout the preparation of this book.

I am also very grateful to all those in Europe, China and Japan who made me welcome, granted me interviews and permitted me to access the documentation, including the Professors Du Shiran, Guo Shuchun, He Shaogeng, Liu Dun, Wang Yusheng, Li Wenlin, Yuan Xiangdong, Pan Jixing and Wu Wenjun (Academia Sinica, Peking), Bai Shangshu (Beijing, Shifan Daxue), Li Di and Luo Jianjin (Univ. Huhehot), Liang Zongju (Univ. Shenyang), Shen Kangshen (Univ. Hangzhou), Wann-Sheng Horng (Taipei, Shifan Daxue) Stanislas Lokuang (Fu-Jen Catholic University), Itō Shuntarō (Tokyo Univ.), Sasaki Chikara, Shimodaira Kazuo (Former President of the Japanese Society for the History of Japanese Mathematics, Tokyo), Murata Tamotsu (Rikkyo Univ., Tokyo), Yoshida Tadashi (Tohoku Univ., Sendai), Hashimoto Keizo, Yabuuchi Kiyoshi (Univ. Kyoto), Joseph Needham and Lu Guizhen (Cambridge), Ullricht Libbrecht (Catholic University, Louvain), Shōkichi Iyanaga, Augustin Berque and Léon Vandermeersch (Maison Franco-Japonaise, Tokyo), René Taton (Centre A. Koyré, Paris), Michel Soymié and Paul Magnin (Institut des Hautes Etudes Chinoises, Dunhuang manuscripts).

I should like to express my thanks to Professors Hirayama Akira (Tokyo), Itagaki Ryōichi (Tokyo), Jiang Zehan (Peking), Christian Houzel (Paris), Adolf Pavlovich Yushkevish (Moscow), Kobayashi Tatsuhiko (Kiryu), Kawahara Hideki (Kyoto), Lam Lay-Yong (Singapore), Edmund Leites (New York), Li Jimin (Xi'an), Guy Mazars (Strasbourg), Yoshimasa Michiwaki (Gunma Univ.), David Mungello (Coe College), Noguchi Taisuke, Ōya Shinichi (Tokyo), Nathan Sivin (Philadelphia), Suzuki Hisao, Tran Van Doan (Fu-Jen Univ., Taipei), Wang Jixun (Suzhou), Yamada Ryōzō (Kyoto) and Joël Brenier (who helped me to enter into contact with Wann-Sheng Horng) (Paris), Khalil Jaouiche (Paris), the late Dr. Shen Shengkun, Mogi Naoko, and Wang Qingxiang.

Finally, I should like to thank the Academia Sinica (Peking), the University of Fu-Jen (Taipei) and the Japanese Society for the Promotion of Science (JSPS, Nihon Gakujutsu Shinkōkai).

March 31st 1987

CNRS, Institut des Hautes
Etudes Chinoises, Paris

Contents

Foreword by J. Gernet	VII
Foreword by J. Dhombres	IX
Preface	XV
Abbreviations	XXIII

Part I. The Context of Chinese Mathematics

1. The Historiographical Context	3
Works on the History of Chinese Mathematics in Western Languages	3
Works on the History of Chinese Mathematics in Japanese	9
Works on the History of Chinese Mathematics in Chinese	10
2. The Historical Context	13
3. The Notion of Chinese Mathematics	41
4. Applications of Chinese Mathematics	47
5. The Structure of Mathematical Works	51
Titles	52
Prefaces	52
Problems	54
Resolutive Rules	58
6. Mathematical Terminology	61
7. Modes of Reasoning	69
8. Chinese Mathematicians	75
9. The Transmission of Knowledge	79
10. Influences and Transmission	89
Possible Contacts with the Seleucids	94
Contacts with India	96
Contacts with Islamic Countries	101
Transmission of Chinese Mathematics to Korea and Japan	105
Contacts with Mongolia	110

Contacts with Tibet	110
Contacts with Vietnam	110
Contacts with Europe	111
11. Main Works and Main Authors (from the Origins to 1600)	123
The Ten Computational Canons	123
Jia Xian and Liu Yi	142
Li Zhi	143
Qin Jiushao	149
Zhu Shijie	152
Yang Hui	157
Cheng Dawei	159
The <i>Suanfa Tongzong</i>	159
The <i>Shuli Jingyun</i>	163
The <i>Chouren Zhuan</i>	166
Li Shanlan	173

Part II. The Content of Chinese Mathematics

12. Numbers and Numeration	179
Knotted Cords (Quipus) and Tallies	179
Chinese Numeration	179
Units of Measurement	191
Fractions, “Models” <i>Lü</i>	192
Decimal Numbers, Metrological and Pure	197
Negative Numbers and Positive Numbers	200
Zero	204
13. Calculating Instruments	209
The Counting Board	209
Counting-Rods	210
The Abacus	211
14. Techniques for Numerical Computation	217
Elementary Operations	217
The Extraction of Roots	221
Systems of Equations of the First Degree in Several Unknowns (Fangcheng Method)	249
13th-Century Chinese Algebra: the <i>Tianyuan Shu</i>	258
15. Geometry	273
Planimetry	277
Stereometry	286
The Right-Angled Triangle	293
Series Summation	302

16. Indeterminate Problems	307
The Hundred Fowls Problem	308
The Remainder Problem	310
17. Approximation Formulae	325
Geometrical Formulae	325
Interpolation Formulae	336
18. Li Shanlan's Summation Formulae	341
19. Infinite Series	353
20. Magic Squares and Puzzles	363
Puzzles	366
Appendix I. Chinese Adaptations of European Mathematical Works (from the 17th to the Beginning of the 19th Century)	371
Appendix II. The Primary Sources	391
Index of Main Chinese Characters	393
(Administrative Terms, Calendars, Geographical Terms, Mathematical Terms, Names of Persons, Other Terms, Titles of Books, Long Expressions)	
References	405
Bibliographical Orientations	405
Books and Articles in Western Languages	407
Books and Articles in Chinese or Japanese	433
Index of Names	463
Index of Books	475
Index of Subjects	481

Abbreviations

CRZ	<i>Chouren zhuan</i> , Taipei (Shijie Shuju, reprinted 1982)
CRZ3B	<i>Chouren zhuan san bian</i> . Ibid.
CRZ4B	<i>Chouren zhuan si bian</i> . Ibid.
CYHJ	<i>Ceyuan haijing</i>
DKW	<i>Dai kanwa jiten</i> by T. Morohashi (Tokyo, 1960)
DicMingBio	<i>Dictionary of Ming Biography (1368–1644)</i> . Goodrich and Fang (1), (eds.), 1976
DSB	<i>Dictionary of Scientific Biography</i> . Gillispie (1), 1970–1980.
HDSJ	<i>Haidao suanjing</i>
Hummel	<i>Eminent Chinese of the Ch'ing Period</i> by A.W. Hummel (reprinted, Taipei (Ch'eng Wen), 1970)
j.	<i>juan</i>
JGSJ	<i>Jigu suanjing</i> (Wang Xiaotong)
JZSS	<i>Jiuzhang suanshu</i>
Li Di, <i>Hist.</i>	<i>Zhongguo shuxue shi jianbian</i> . Li Di (3'), 1984
Li Yan, <i>Dagang</i>	<i>Zhongguo shuxue dagang</i> . Li Yan (56'), 1958
Li Yan, <i>Gudai</i>	<i>Zhongguo gudai shuxue shiliao</i> . Li Yan (61'), 1954/1963
<i>Meijizen</i>	<i>Meijizen Nihon sūgaku shi</i> Nihon Gakushin (1'), 1954–60
MSCSJY	<i>Meishi congshu jiyao</i> . Mei Zuangao ed., 1874
Nine Chapters	<i>Jiuzhang suanshu</i>
QB	<i>Suanjing shishu</i> . Qian Baocong (25'), 1963
QB, <i>Hist.</i>	<i>Zhongguo shuxue shi</i> . Qian Baocong, (26'), 1964
RBS	<i>Revue Bibliographique de Sinologie</i> (Paris)
SCC	<i>Science and Civilisation in China</i> . Needham (2), 1959
SFTZ	<i>Suanfa tongzong</i> (Cheng Dawei, 1592)
SJSS	<i>Suanjing shi shu</i> (Ten Computational Classics)
SLJY	<i>Shuli jingyun</i> (1723)
SSJZ	<i>Shushu jiuzhang</i> (Qin Jiushao, 1247)
SXQM	<i>Suanxue qimeng</i> (Zhu Shijie, 1299)
SY	<i>Song Yuan shuxue shi lunwen ji</i> , Qian Baocong et al. (1'), 1966
SYYY	<i>Siyuan yujian</i> (Zhu Shijie, 1303)
SZSJ	<i>Sunzi suanjing</i>
Wang Ling, <i>Thesis</i>	Wang Ling (1), 1956
WCSJ	<i>Wucaosuanjing</i>
XHYSJ	<i>Xiahou Yang suanjing</i>
YLDD	<i>Yongle dadian</i>
ZBSJ	<i>Zhoubi suanjing</i>

- ZQJSJ *Zhang Qiuqian suanjing*
 ZSSLC-P *Zhong suan shi luncong*. Li Yan (51'), 1954–1955
 ZSSLC-T *Zhong suan shi luncong*. Li Yan (41'), 1937/1977

Remarks

- An abbreviation such as *JZSS* 7-2 refers to problem number 2 of chapter 7 of the *Jiuzhang suanshu* (or to the commentary to that problem).
- DKW 10-35240: 52, p. 10838 refers the entry number 52 corresponding to the Chinese written character number 35240 in volume 10 of the *Dai kanwa jiten* (Great Chinese–Japanese Dictionary) by MOROHASHI Tetsuji (Tokyo, 1960), page 10838.
- Pages numbers relating to the twenty-four Standard Histories always refer to the edition of the text published by Zhonghua Shuju (Peking) from 1965.
- Certain references to works cited in the bibliographies concern reprinted works. In such a case, as far as possible, the bibliography mentions two years of publication, that of the first edition and that of the reprint. Unless otherwise stated, all mentions of pages concerning such works always refer to the reprint. For example, “GRANET Marcel (1), 1934/1968. *La Pensée Chinoise*. Paris: Albin Michel” is cited as “Granet (1), 1934” but the pages mentioned in the footnotes concern the 1968 reprint of this work.

Author's Note

The present English translation is a revised and augmented version of my *Histoire des mathématiques chinoises*, Paris, Masson, 1987. New chapters have been added and the bibliography has been brought up to date. I express my thanks to the translator, Dr. Stephen S. Wilson, and to the staff of Springer, particularly Dr. Catriona C. Byrne, Ingrid Beyer and Kerstin Graf. I am also much indebted to Mr. Karl-Friedrich Koch for his careful collaboration and professionalism. Mr. Olivier Gérard has been helpful at the early stage of the composition of the book. Last but not least, many thanks to Ginette Kotowicz, Nicole Resche and all the librarians of the Institut des Hautes Etudes Chinoises, Paris.