

George E. Martin

Transformation Geometry

An Introduction to Symmetry

With 222 Illustrations



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