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FRACTALS AND CHAOS

The Mandelbrot Set
and Beyond

SELECTA VOLUME C

*with a foreword by P.W. Jones
and texts co-authored by
C.J.G. Evertsz and M.C. Gutzwiller*



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