

Benoit B. Mandelbrot

FRACTALS *and*  
SCALING  
*in* FINANCE

*Discontinuity,  
Concentration, Risk*

SELECTA VOLUME E

*With Foreword by R.E. Gomory  
and Contributions by P.H. Cootner, E.F. Fama,  
W.S. Morris, H.M. Taylor, and others*

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