# HIGH-DIMENSIONAL NONLINEAR DIFFUSION STOCHASTIC PROCESSES

**Modelling for Engineering Applications** 

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