Undergraduate Texts in Mathematics

Jerome Malitz

Introduction to Mathematical Logic

Set Theory
Computable Functions
Model Theory

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For Sue, Jed, and Seth

Contents

Preface

This book is intended as an undergraduate senior level or beginning graduate level text for mathematical logic. There are virtually no prerequisites, although a familiarity with notions encountered in a beginning course in abstract algebra such as groups, rings, and fields will be useful in providing some motivation for the topics in Part III.

An attempt has been made to develop the beginning of each part slowly and then to gradually quicken the pace and the complexity of the material. Each part ends with a brief introduction to selected topics of current interest.

The text is divided into three parts: one dealing with set theory, another with computable function theory, and the last with model theory. Part III relies heavily on the notation, concepts and results discussed in Part I and to some extent on Part II. Parts I and II are independent of each other, and each provides enough material for a one semester course.

The exercises cover a wide range of difficulty with an emphasis on more routine problems in the earlier sections of each part in order to familiarize the reader with the new notions and methods. The more difficult exercises are accompanied by hints. In some cases significant theorems are developed step by step with hints in the problems. Such theorems are not used later in the sequence.

The part dealing with set theory is intended to provide a notational and conceptual framework for areas of mathematics outside of logic as well as to introduce the student to those topics that are of particular interest to those working in the foundations of set theory.

We hope that the part of the text devoted to computable functions will be of interest to those who intend to work with real world computers.

We believe that the notation, methodology, and results of elementary logic should be a part of a general mathematics program and are of value in a wide variety of disciplines within mathematics and outside of mathematics.

Boulder, Colorado
March 1979

J. MALITZ

Glossary of Symbols

$\{\cdots\}$	2
$\mathbf{I}$	2
$\mathbf{N}$	2
$\mathbf{N}^+$	2
$\mathbf{Q}$	2
$\mathbf{Q}^+$	2
$\mathbf{R}$	2
$\mathbf{R}^+$	2
$\{x:\cdots\}$	2
$\varnothing$	2
$\in$	2
$\subseteq, \supseteq, \subset, \supset$	2, 3
$\cup$	3
$\bigcup X$	4
$\bigcup_{i\in I} A_i$	4
$\cap$	4
$\bigcap X$	4
$B-A$	4
$P(X)$	5
$A\times B$	6
$[B]_k$	6
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Ranz R	7
$1-1$	7
$f:A\rightarrow B$	7
${}^{A}B$	7
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$\forall x < y$	75
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Prime	75

Prim	75
Exp′	76
Max	77
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$\dot{n}$	80
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