SERIES IN OPTICS AND OPTOELECTRONICS

Laser-Based Measurements for Time and Frequency Domain Applications

A Handbook

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Contents

Foreword xv

Preface xvii

Authors xix

1 Shedding light on the art of timekeeping 1

1.1 The great show of Time and Light, the curtain rises! 1
1.2 Brief history of timekeeping: time-frequency equivalence 2
1.3 The parallel story of the speed of light 6
  1.3.1 The laser arrives: length-frequency equivalence and the birth of optical frequency metrology 14
  1.3.2 Role of c in fundamental physics 17
1.4 In the end, time and light met up again: optical atomic clocks and outline of the book 20

2 Characterization and control of harmonic oscillators 25

2.1 The ideal harmonic oscillator 25
  2.1.1 Synchronization in coupled oscillators 32
  2.1.2 Beating two oscillators 35
2.2 Self-sustained oscillators 36
2.3 The noisy oscillator 41
2.4 Phase noise 41
  2.4.1 Review of mathematical tools 42
  2.4.2 Fundamental noise mechanisms 44
    2.4.2.1 Fluctuation-dissipation theorem (FDT) 48
2.5 Phase noise modelling 51
  2.5.1 The Leeson effect 54
  2.5.2 The Allan variance 57
  2.5.3 Oscillator power spectrum in the carrier frequency domain 62
    2.5.3.1 Effect of frequency multiplication on the power spectrum 65
2.6 Noise reduction in oscillators 66
  2.6.1 Phase-locked loops 66
    2.6.1.1 Optical phase-locked loops 74
  2.6.2 Injection locking 76
2.7 Phase noise measurements 80
  2.7.1 Frequency counting 80
  2.7.2 Homodyne techniques 81
  2.7.3 Heterodyne techniques 82
  2.7.4 Self-heterodyning 83

2.8 Amplitude noise measurements 84
  2.8.1 AM noise in optical systems 85

3 Passive resonators 87

3.1 Microwave cavities 88
3.2 Basic properties of bulk optical cavities 94
  3.2.1 Fabry-Perot etalon 95
  3.2.2 Paraxial ray analysis 101
  3.2.3 Wave analysis 102
3.3 Cavity-design considerations 106
  3.3.1 Quarter wave stack reflectors 107
  3.3.2 Prism-based cavities 111
3.4 Ultrastable cavities 117
  3.4.1 Thermal stability 117
  3.4.2 Vibration insensitive optical cavities 123
3.5 Fiber cavities 128
  3.5.1 Directional coupler-based fiber cavities 129
  3.5.2 Resonators based on closely faced fiber tips 131
  3.5.3 FBG-based fiber resonators 132
3.6 Whispering gallery mode resonators 137
  3.6.1 Wave theory of whispering gallery modes 137
  3.6.2 WGMs in a ray-optical picture 150
  3.6.3 Mode Q and volume 152
  3.6.4 WGM evanescent coupling 154
  3.6.5 Fabrication and applications of whispering gallery resonators 159

4 Continuous-wave coherent radiation sources 163

4.1 Principles of masers 164
  4.1.1 The hydrogen maser 171
4.2 Compendium of laser theory 176
  4.2.1 The active medium 177
    4.2.1.1 Einstein A and B coefficients for absorption and emission . 177
    4.2.1.2 Line-broadening mechanisms 180
  4.2.2 The pump 183
  4.2.3 The resonator 186
  4.2.4 Rate equations for a four-level system 188
    4.2.4.1 Transient behavior and relaxation oscillation 191
4.3 Frequency pulling 193
4.4 Achieving single-mode oscillation 194
  4.4.1 Line selection 197
  4.4.2 Single-tranverse mode selection 198
  4.4.3 Single-longitudinal mode selection 198
4.5 The laser output 202
  4.5.1 Spatial coherence 202
  4.5.2 Spectral and temporal coherence 203
  4.5.3 The effect of spontaneous emission 205
    4.5.3.1 Intrinsic laser linewidth 206
    4.5.3.2 Amplified spontaneous emission 211
4.6 Laser frequency fluctuations and stabilization techniques 213

4.6.1 Side-lock to an atomic/molecular resonance . . . 219
4.6.2 Pound-Drever-Hall method . . . 220
4.6.2.1 10^{-16}-level laser frequency stabilization . . . 225
4.6.2.2 WGM resonators for laser frequency stabilization . . . 228
4.6.3 Hänsch-Couillaud technique . . . 231
4.6.4 Laser frequency stabilization by locking to an optical fiber-delay line 233
4.6.5 Injection locking . . . 237
4.7 Intensity fluctuations . . . 240
4.7.1 High-sensitivity photodiode array . . . 243
4.7.2 Optical ac coupling . . . 244
4.7.3 The laser as quasi-ideal oscillator . . . 246
4.8 Some specific laser systems . . . 249
4.8.1 He-Ne laser . . . 250
4.8.2 Carbon dioxide laser . . . 251
4.8.3 Dye lasers . . . 253
4.8.4 Ion-doped lasers and optical amplifiers . . . 257
4.8.4.1 Nd:YAG laser . . . 257
4.8.4.2 Ti:Sa laser . . . 258
4.8.4.3 Erbium-doped fiber amplifiers (EDFAs) . . . 259
4.8.4.4 Ytterbium-doped fiber amplifiers (YDFAs) . . . 260
4.8.4.5 Narrow-linewidth fiber lasers . . . 263
4.8.5 Semiconductor lasers . . . 268
4.8.5.1 Heterostructure diode lasers . . . 269
4.8.5.2 Distributed feedback (DFB) lasers . . . 285
4.8.5.3 Tapered semiconductor amplifiers . . . 288
4.8.5.4 Multiple-quantum-well lasers . . . 289
4.8.5.5 Vertical-cavity surface-emitting lasers . . . 292
4.8.5.6 Quantum cascade lasers . . . 292
4.8.5.7 Interband cascade lasers . . . 300
4.8.6 Nonlinear laser cw sources . . . 302
4.8.6.1 Sum frequency generation . . . 305
4.8.6.2 Optical parametric oscillators . . . 306
4.8.6.3 Difference frequency generators . . . 312
4.8.6.4 Tunable far-infrared radiation . . . 317

5 High-resolution spectroscopic frequency measurements 323

5.1 Interferometric wavelength measurements . . . 323
5.2 Spectroscopic frequency measurements . . . 328
5.2.1 Principles of absorption laser spectroscopy . . . 329
5.3 Frequency modulation spectroscopy . . . 329
5.3.1 Harmonic detection . . . 330
5.3.2 Wavelength modulation spectroscopy . . . 331
5.3.3 Single- and two-tone frequency modulation spectroscopy . . . 331
5.4 Magnetic rotation spectroscopy . . . 337
5.5 Cavity-enhanced spectroscopy . . . 341
5.5.1 Cavity-enhanced absorption spectroscopy . . . 342
5.5.2 Off-axis integrated cavity output spectroscopy . . . 343
5.5.3 Noise immune cavity-enhanced optical heterodyne molecular spectroscopy . . . 346
5.5.4 Cavity ring-down spectroscopy . . . 348

5.5.4.1 Phase-shift (PS) CRDS 350
5.5.4.2 Saturated-absorption cavity ring-down spectroscopy 351
5.6 Doppler-free saturation spectroscopy 354
5.6.1 Frequency locking to a Lamb dip 358
5.6.2 Cavity-enhanced Doppler-free saturation spectroscopy 361
5.6.2.1 Doppler-free NICE-OHMS 363
5.7 Doppler-free polarization spectroscopy 364
5.8 Doppler-free two-photon spectroscopy 368
5.9 Second-order Doppler-free spectroscopy 372
5.10 Sub-Doppler spectroscopy in atomic/molecular beams 374
5.10.1 Effusive beams 374
5.10.2 Supersonic beams 377
5.10.3 Buffer-gas-cooling 379
5.11 Ramsey fringes 381
5.12 Laser frequency standards using thermal quantum absorbers 387
5.12.1 Iodine-stabilized lasers 388
5.12.2 Acetylene-stabilized lasers 388
5.12.3 Methane-stabilized lasers 389
5.12.4 OsO_4-stabilized lasers 389
5.12.5 Atomic hydrogen standard 390
5.12.6 Calcium standard 393
5.13 Fourier transform spectroscopy 394
5.14 Raman spectroscopy 398
5.14.1 Coherent anti-Stokes spectroscopy 405
5.14.2 Stimulated Raman scattering 410

6 Time and frequency measurements with pulsed laser systems 413

6.1 Introduction 413
6.2 Theory of mode locking 415
6.3 Mode-locking mechanisms and dispersion compensation schemes 417
6.3.1 Ti:sapphire lasers and Kerr-lens mode locking 417
6.3.2 Fiber-based lasers and nonlinear-polarization-rotation mode-locking 419
6.4 Optical frequency comb synthesis from mode-locked lasers 421
6.4.1 Comb stabilization 421
6.4.2 Measurements with a frequency comb 425
6.4.2.1 Measuring frequency differences 425
6.4.2.2 Measuring absolute frequencies with an octave-spanning comb 426
6.4.2.3 Absolute optical frequency synthesizer 427
6.4.2.4 Direct frequency-comb spectroscopy 429
6.4.2.5 Other measurement schemes and applications 435
6.4.3 Relevant properties of a mode-locked laser for frequency-comb applications 440
6.4.4 Microresonator-based frequency combs 441
6.5 Extension of OFCSs into novel spectral regions 444
6.5.1 High-order laser harmonics and extensions to the XUV 444
6.5.1.1 Generation and properties of high-order laser harmonics . . 446
6.5.1.2 Ramsey-type spectroscopy in the XUV with high-order harmonics 448
6.5.1.3 Cavity-enhanced XUV frequency combs 456

6.5.1.4 Attosecond pulses . . . 459
6.5.2 Mid- and far-infrared OFCSs . . . 460

7 Frequency standards 465

7.1 General features of frequency standards and clocks . . . 465
7.2 Quartz oscillators . . . 466
7.2.1 Factors affecting crystal oscillator frequency accuracy . . . 471
7.2.2 Factors affecting frequency stability . . . 473
7.2.3 State-of-the-art ultrastable quartz oscillators . . . 473
7.3 Cryogenic sapphire oscillators . . . 473
7.4 Photonic microwave oscillators based on WGM resonators . . . 479
7.5 Generation of ultrastable microwaves via optical frequency division . . . 480
7.6 Trapping and cooling of neutral atoms . . . 482
7.6.1 Optical molasses . . . 483
7.6.2 Magneto-optical traps . . . 489
7.6.3 Bose-Einstein condensation . . . 492
7.6.3.1 Magnetic trapping . . . 492
7.6.3.2 Evaporative cooling . . . 493
7.6.3.3 Probing a BEC . . . 495
7.7 Cold stable molecules . . . 498
7.7.1 Stark decelerator . . . 498
7.8 Trapping and cooling of ions . . . 500
7.8.1 Paul traps . . . 500
7.8.1.1 Linear Paul traps . . . 504
7.8.2 Penning traps . . . 505
7.8.3 Trap loading . . . 507
7.8.4 Ion cooling techniques . . . 507
7.8.4.1 Laser cooling . . . 508
7.8.4.2 Sympathetic cooling . . . 511
7.8.5 Spectroscopy of trapped particles in the Lamb-Dicke regime . . . 511
7.9 Microwave atomic standards . . . 513
7.9.1 Metrological properties of the active hydrogen maser . . . 513
7.9.1.1 Maser design . . . 513
7.9.1.2 Frequency shifts . . . 513
7.9.1.3 Automatic tuning of the resonant cavity . . . 515
7.9.1.4 Frequency stability . . . 516
7.9.1.5 Cryogenic hydrogen masers . . . 517
7.9.2 Cesium clocks . . . 518
7.9.2.1 Cesium-beam frequency standards . . . 518
7.9.2.2 Cesium fountain clocks . . . 521
7.9.2.3 Uncertainty budget in a Cs fountain clock . . . 527
7.9.3 Rubidium clocks . . . 532
7.9.3.1 Rb fountain clocks . . . 533
7.9.3.2 Lamp-based Rb cell standards . . . 534
7.9.3.3 Laser-based Rb cell frequency standards . . . 535
7.9.4 Microwave ion clocks . . . 539
7.10 Time transfer and frequency dissemination . . . 541
7.10.1 Realization of time scales . . . 542
7.10.1.1 Realization of TAI . . . 542

7.10.1.2 Coordinated universal time 543
7.10.2 Transmitting time information 543
7.10.2.1 Portable clocks 546
7.10.2.2 Global positioning system 547
7.10.3 Frequency transfer 550
7.10.3.1 A democratic absolute frequency chain 550
7.10.3.2 Dissemination of microwave frequency standards 551
7.10.3.3 Optical frequency transfer 552

8 Future trends in fundamental physics and applications 557

8.1 Optical atomic clocks 557
8.1.1 Trapped ion optical clocks 559
8.1.1.1 Systematic frequency shifts 562
8.1.2 Neutral atoms optical lattice clocks 564
8.2 The hydrogen atom as an inexhaustible wellspring of advances in precision spectroscopy 570
8.2.1 Determination of the Rydberg constant and of the proton radius . . 572
8.3 Spectroscopy of cold, trapped metastable helium 575
8.4 Measurements of fundamental constants 578
8.4.1 Boltzmann constant k_B 578
8.4.2 Newton gravitational constant G 580
8.5 Constancy of fundamental constants 582
8.5.1 Fine structure constant α 584
8.5.1.1 Proton-to-electron mass ratio β 586
8.5.2 Speed of light c 588
8.5.2.1 Frequency dependence of c and the mass of the photon . . 588
8.5.3 Newton's gravitational constant 590
8.6 Tests of fundamental physics laws 590
8.6.1 Spectroscopic tests of spin-statistic connection and symmetrization postulate 590
8.6.2 Search for the electron dipole moment 594
8.6.3 Parity violation in chiral molecules 595
8.7 Perspectives for precision spectroscopy of cold molecules 600
8.8 Tests of general relativity: from ground-based experiments to space missions 603
8.8.1 Testing the Einstein Equivalence Principle 604
8.8.1.1 Tests of UFF 604
8.8.1.2 Tests of LLI 605
8.8.1.3 Tests of LPI 608
8.8.2 Test of post-Newtonian gravity 610
8.8.3 Tests of the gravitational inverse square law 611
8.8.3.1 Detection of gravitational waves 611
8.9 Quantum-enhanced time and frequency measurements 613
8.9.1 Standard quantum limit in physical measurements 613
8.9.2 Using nonclassical light states for quantum-enhanced measurements 615
8.9.3 Applications to time and frequency measurements 620
8.9.3.1 Quantum logic spectroscopy 620
8.9.3.2 Time and frequency quantum metrology 621
8.9.3.3 Quantum positioning and clock synchronization 623
8.10 Environmental metrology 625

8.10.1 Geophysical survey of volcanic areas 625
8.10.2 Detection of very rare isotopes 628
8.10.3 Stratospheric survey with tunable diode laser spectrometers 628
8.10.4 Fiber sensing of physical and chemical parameters 630
8.10.4.1 Strain sensing 632
8.10.4.2 Acceleration measurements 633
8.10.4.3 Chemical sensing by optical-fiber ring resonators 634

Bibliography **637**

Index **701**