Saunders Mac Lane

Homology

Reprint of the 1975 Edition



Saunders Mac Lane Department of Mathematics, University of Chicago Chicago, IL 60637-1514 USA

Originally published as Vol. 114 of the Grundlehren der mathematischen Wissenschaften

Mathematics Subject Classification (1991): 18-02, 18AXX, 18CXX, 18GXX

ISBN-13: 978-3-540-58662-3 DOI: 10.1007/978-3-642-62029-4 e-ISBN-13: 978-3-642-62029-4

CIP data applied for

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustration, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provision of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1995

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

41/3111 - 5 4 3 2 1 Printed on acid-free paper

Table of Contents

1. The Arrow Notation	8 9 13 15
2. Modules	9 13
	13
3 Diagrams	-
J. Diagrams	15
4. Direct Sums	• •
	19
6. The Functor Hom	21
	25
8. Functors	28
Chapter II. Homology of Complexes	34
1. Differential Groups	35
-	39
1	42
	44
	49
	51
	54
	57
	61
	63
	05
1. Extensions of Modules	63
2. Addition of Extensions	67
· · · · · · · · · · · · · · · · · · ·	72
4. The Universal Coefficient Theorem for Cohomology	76
5. Composition of Extensions	82
	87
	92
8. Injective Resolutions	95
9. Two Exact Sequences for Ext^n	96
	99
11. The Injective Envelope	02
Chapter IV. Cohomology of Groups	03
1. The Group Ring	04
2. Crossed Homomorphisms	05
3. Group Extensions	
4. Factor Sets	
5. The Bar Resolution	
6. The Characteristic Class of a Group Extension	20

Table of Contents

pag	
7. Cohomology of Cyclic and Free Groups	
8. Obstructions to Extensions	
9. Realization of Obstructions	-
10. Schur's Theorem	
11. Spaces with Operators	4
Chapter V. Tensor and Torsion Products	8
1. Tensor Products	8
2. Modules over Commutative Rings	
3. Bimodules	
4. Dual Modules	
	-
5. Right Exactness of Tensor Products	-
6. Torsion Products of Groups	
7. Torsion Products of Modules	
8. Torsion Products by Resolutions	
9. The Tensor Product of Complexes	
10. The KÜNNETH Formula	6
11. Universal Coefficient Theorems	0
Chapter VI. Types of Algebras	3
1. Algebras by Diagrams	3
2. Graded Modules	5
3. Graded Algebras	7
4. Tensor Products of Algebras	1
5. Modules over Algebras	
6. Cohomology of free Abelian Groups	
7. Differential Graded Algebras	
	-
8. Identities on Hom and \otimes	
9. Coalgebras and HOPF Algebras	1
Chapter VII. Dimension	0
1. Homological Dimension	0
2. Dimensions in Polynomial Rings	
3. Ext and Tor for Algebras	
4. Global Dimensions of Polynomial Rings	
5. Separable Algebras	
6. Graded Syzygies	
7. Local Rings	0
Chapter VIII. Products	0
1. Homology Products	
2. The Torsion Product of Algebras 2.	
3. A Diagram Lemma	
4. External Products for Ext	
5. Simplicial Objects	
6. Normalization	
7. Acyclic Models	
8. The Eilenberg-Zilber Theorem	
9. Cup Products	.4
Chapter IX. Relative Homological Algebra	
1. Additive Categories	9
2. Abelian Categories	64

IX

Table of Contents

page	9
3. Categories of Diagrams	
4. Comparison of Allowable Resolutions	
5. Relative Abelian Categories	
6. Relative Resolutions	
7. The Categorical Bar Resolution	
8. Relative Torsion Products	-
9. Direct Products of Rings	3
Chapter X. Cohomology of Algebraic Systems	n
1. Introduction	
0	
3. The Cohomology of an Algebra	
4. The Homology of an Algebra	
5. Homology of Groups and Monoids	
6. Ground Ring Extensions and Direct Products	-
7. Homology of Tensor Products	-
8. The Case of Graded Algebras	
9. Complexes of Complexes	
10. Resolutions and Constructions	-
11. Two-stage Cohomology of DGA-Algebras	
12. Cohomology of Commutative DGA-Algebras	
13. Homology of Algebraic Systems	5
Chapter XI. Spectral Sequences	8
1. Spectral Sequences	
4. Transgression	
5. Exact Couples	
6. Bicomplexes	
7. The Spectral Sequence of a Covering	
8. Cohomology Spectral Sequences	-
9. Restriction, Inflation, and Connection	
10. The Lyndon Spectral Sequence	
11. The Comparison Theorem	7
Chapter XII. Derived Functors	8
1. Squares	-
2. Subobjects and Quotient Objects	
3. Diagram Chasing	
4. Proper Exact Sequences	•
5. Ext without Projectives $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 37$	
6. The Category of Short Exact Sequences	5
7. Connected Pairs of Additive Functors	9
8. Connected Sequences of Functors	6
9. Derived Functors	9
10. Products by Universality	4
11. Proper Projective Complexes	7
12. The Spectral KUNNETH Formula	0
Bibliography	4
List of Standard Symbols	3
Index	5