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# Categories for the Working Mathematician

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# Preface

Category Theory has developed rapidly. This book aims to present those ideas and methods which can now be effectively used by Mathematicians working in a variety of other fields of Mathematical research. This occurs at several levels. On the first level, categories provide a convenient conceptual language, based on the notions of category, functor, natural transformation, contravariance, and functor category. These notions are presented, with appropriate examples, in Chapters I and II. Next comes the fundamental idea of an adjoint pair of functors. This appears in many substantially equivalent forms: That of universal construction, that of direct and inverse limit, and that of pairs of functors with a natural isomorphism between corresponding sets of arrows. All these forms, with their interrelations, are examined in Chapters III to V. The slogan is "Adjoint functors arise everywhere".

Alternatively, the fundamental notion of category theory is that of a monoid—a set with a binary operation of multiplication which is associative and which has a unit; a category itself can be regarded as a sort of generalized monoid. Chapters VI and VII explore this notion and its generalizations. Its close connection to pairs of adjoint functors illuminates the ideas of universal algebra and culminates in Beck's theorem characterizing categories of algebras; on the other hand, categories with a monoidal structure (given by a tensor product) lead *inter alia* to the study of more convenient categories of topological spaces.

Since a category consists of arrows, our subject could also be described as learning how to live without elements, using arrows instead. This line of thought, present from the start, comes to a focus in Chapter VIII, which covers the elementary theory of abelian categories and the means to prove all the diagram lemmas without ever chasing an element around a diagram.

Finally, the basic notions of category theory are assembled in the last two chapters: More exigent properties of limits, especially of filtered limits, a calculus of "ends", and the notion of Kan extensions. This is the deeper form of the basic constructions of adjoints. We end with the observations that all concepts of category theory are Kan extensions (§ 7 of Chapter X).

I have had many opportunities to lecture on the materials of these chapters: At Chicago; at Boulder, in a series of Colloquium lectures to the American Mathematical Society; at St. Andrews, thanks to the Edinburgh Mathematical Society; at Zurich, thanks to Beno Eckmann and the Forschungsinstitut für Mathematik; at London, thanks to A. Fröhlich and Kings and Queens Colleges; at Heidelberg, thanks to H. Seifert and Albrecht Dold; at Canberra, thanks to Neumann, Neumann, and a Fulbright grant; at Bowdoin, thanks to Dan Christie and the National Science Foundation; at Tulane, thanks to Paul Mostert and the Ford Foundation, and again at Chicago, thanks ultimately to Robert Maynard Hutchins and Marshall Harvey Stone.

Many colleagues have helped my studies. I have profited much from a succession of visitors to Chicago (made possible by effective support from the Air Force Office of Scientific Research, the Office of Naval Research, and the National Science Foundation): M. André, J. Bénabou, E. Dubuc, F. W. Lawvere, and F. E. J. Linton. I have had good counsel from Michael Barr, John Gray, Myles Tierney, and Fritz Ulmer, and sage advice from Brian Abrahamson, Ronald Brown, W. H. Cockcroft, and Paul Halmos. Daniel Feigin and Geoffrey Phillips both managed to bring some of my lectures into effective written form. My old friend, A. H. Clifford, and others at Tulane were of great assistance. John MacDonald and Ross Street gave pertinent advice on several chapters; Spencer Dickson, S. A. Huq, and Miguel La Plaza gave a critical reading of other material. Peter May's trenchant advice vitally improved the emphasis and arrangement, and Max Kelly's eagle eye caught many soft spots in the final manuscript. I am grateful to Dorothy Mac Lane and Tere Shuman for typing, to Dorothy Mac Lane for preparing the index and to M. K. Kwong for careful proof reading – but the errors which remain, and the choice of emphasis and arrangement, are mine.

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