

An Invitation to 3-D Vision

From Images to Geometric Models

Yi Ma
Stefano Soatto
Jana Kořecká
S. Shankar Sastry

With 170 Illustrations



Springer Science+Business Media, LLC

Yi Ma
Department of Electrical and
Computer Engineering
University of Illinois at
Urbana-Champaign
Urbana, IL 61801
USA
yima@uiuc.edu

Jana Košecká
Department of Computer Science
George Mason University
Fairfax, VA 22030
USA
kosecka@cs.gmu.edu

Editors

S.S. Antman
Department of Mathematics
and
Institute for Physical Science
and Technology
University of Maryland
College Park, MD 20742
USA
ssa@math.umd.edu

L. Sirovich
Division of Applied Mathematics
Brown University
Providence, RI 02912
USA
chico@camelot.mssm.edu

Stefano Soatto
Department of Computer Science
University of California, Los Angeles
Los Angeles, CA 90095
USA
soatto@ucla.edu

S. Shankar Sastry
Department of Electrical Engineering
and Computer Science
University of California, Berkeley
Berkeley, CA 94720
USA
sastry@eecs.berkeley.edu

J.E. Marsden
Control and Dynamical Systems
Mail Code 107-81
California Institute of Technology
Pasadena, CA 91125
USA
marsden@cds.caltech.edu

S. Wiggins
School of Mathematics
University of Bristol
Bristol BS8 1TW
UK
s.wiggins@bris.ac.uk

Cover Illustration: "Axo-GJ" 1968 (180 × 180 cm) by Victor Vasarely. Copyright Michèle Vasarely.

Mathematics Subject Classification (2000): 51U10, 68U10, 65D18

ISBN 978-1-4419-1846-8 ISBN 978-0-387-21779-6 (eBook)

DOI 10.1007/978-0-387-21779-6

Printed on acid-free paper.

© 2004 Springer Science+Business Media New York

Originally published by Springer-Verlag New York, Inc. in 2004

Softcover reprint of the hardcover 1st edition 2004

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher, (Springer Science+Business Media New York) except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

9 8 7 6 5 4 3 2 (EB)

springeronline.com

Contents

Preface	vii
Acknowledgments	xi
1 Introduction	1
1.1 Visual perception from 2-D images to 3-D models	1
1.2 A mathematical approach	8
1.3 A historical perspective	9
I Introductory Material	13
2 Representation of a Three-Dimensional Moving Scene	15
2.1 Three-dimensional Euclidean space	16
2.2 Rigid-body motion	19
2.3 Rotational motion and its representations	22
2.3.1 Orthogonal matrix representation of rotations	22
2.3.2 Canonical exponential coordinates for rotations	25
2.4 Rigid-body motion and its representations	28
2.4.1 Homogeneous representation	29
2.4.2 Canonical exponential coordinates for rigid-body motions	31
2.5 Coordinate and velocity transformations	34
2.6 Summary	37

2.7	Exercises	38
2.A	Quaternions and Euler angles for rotations	40
3	Image Formation	44
3.1	Representation of images	46
3.2	Lenses, light, and basic photometry	47
3.2.1	Imaging through lenses	48
3.2.2	Imaging through a pinhole	49
3.3	A geometric model of image formation	51
3.3.1	An ideal perspective camera	52
3.3.2	Camera with intrinsic parameters	53
3.3.3	Radial distortion	58
3.3.4	Image, preimage, and coimage of points and lines	59
3.4	Summary	62
3.5	Exercises	62
3.A	Basic photometry with light sources and surfaces	65
3.B	Image formation in the language of projective geometry	70
4	Image Primitives and Correspondence	75
4.1	Correspondence of geometric features	76
4.1.1	From photometric features to geometric primitives	77
4.1.2	Local vs. global image deformations	78
4.2	Local deformation models	80
4.2.1	Transformations of the image domain	80
4.2.2	Transformations of the intensity value	82
4.3	Matching point features	82
4.3.1	Small baseline: feature tracking and optical flow	84
4.3.2	Large baseline: affine model and normalized cross-correlation	88
4.3.3	Point feature selection	90
4.4	Tracking line features	92
4.4.1	Edge features and edge detection	93
4.4.2	Composition of edge elements: line fitting	94
4.4.3	Tracking and matching line segments	95
4.5	Summary	96
4.6	Exercises	97
4.A	Computing image gradients	99
II	Geometry of Two Views	107
5	Reconstruction from Two Calibrated Views	109
5.1	Epipolar geometry	110
5.1.1	The epipolar constraint and the essential matrix	110
5.1.2	Elementary properties of the essential matrix	113

5.2	Basic reconstruction algorithms	117
5.2.1	The eight-point linear algorithm	117
5.2.2	Euclidean constraints and structure reconstruction . .	124
5.2.3	Optimal pose and structure	125
5.3	Planar scenes and homography	131
5.3.1	Planar homography	131
5.3.2	Estimating the planar homography matrix	134
5.3.3	Decomposing the planar homography matrix	136
5.3.4	Relationships between the homography and the essential matrix	139
5.4	Continuous motion case	142
5.4.1	Continuous epipolar constraint and the continuous essential matrix	142
5.4.2	Properties of the continuous essential matrix	144
5.4.3	The eight-point linear algorithm	148
5.4.4	Euclidean constraints and structure reconstruction . .	152
5.4.5	Continuous homography for a planar scene	154
5.4.6	Estimating the continuous homography matrix	155
5.4.7	Decomposing the continuous homography matrix . .	157
5.5	Summary	158
5.6	Exercises	159
5.A	Optimization subject to the epipolar constraint	165
6	Reconstruction from Two Uncalibrated Views	171
6.1	Uncalibrated camera or distorted space?	174
6.2	Uncalibrated epipolar geometry	177
6.2.1	The fundamental matrix	177
6.2.2	Properties of the fundamental matrix	178
6.3	Ambiguities and constraints in image formation	181
6.3.1	Structure of the intrinsic parameter matrix	182
6.3.2	Structure of the extrinsic parameters	184
6.3.3	Structure of the projection matrix	184
6.4	Stratified reconstruction	185
6.4.1	Geometric stratification	185
6.4.2	Projective reconstruction	188
6.4.3	Affine reconstruction	192
6.4.4	Euclidean reconstruction	194
6.4.5	Direct stratification from multiple views (preview) . .	196
6.5	Calibration with scene knowledge	198
6.5.1	Partial scene knowledge	199
6.5.2	Calibration with a rig	201
6.5.3	Calibration with a planar pattern	202
6.6	Dinner with Kruppa	204
6.7	Summary	206
6.8	Exercises	206

6.A	From images to fundamental matrices	211
6.B	Properties of Kruppa's equations	215
6.B.1	Linearly independent Kruppa's equations under special motions	217
6.B.2	Cheirality constraints	223
7	Estimation of Multiple Motions from Two Views	228
7.1	Multibody epipolar constraint and the fundamental matrix . .	229
7.2	A rank condition for the number of motions	234
7.3	Geometric properties of the multibody fundamental matrix .	237
7.4	Multibody motion estimation and segmentation	242
7.4.1	Estimation of epipolar lines and epipoles	243
7.4.2	Recovery of individual fundamental matrices	247
7.4.3	3-D motion segmentation	248
7.5	Multibody structure from motion	249
7.6	Summary	252
7.7	Exercises	253
7.A	Homogeneous polynomial factorization	256

III Geometry of Multiple Views 261

8	Multiple-View Geometry of Points and Lines	263
8.1	Basic notation for the (pre)image and coimage of points and lines	264
8.2	Preliminary rank conditions of multiple images	267
8.2.1	Point features	267
8.2.2	Line features	270
8.3	Geometry of point features	273
8.3.1	The multiple-view matrix of a point and its rank . . .	273
8.3.2	Geometric interpretation of the rank condition	276
8.3.3	Multiple-view factorization of point features	278
8.4	Geometry of line features	283
8.4.1	The multiple-view matrix of a line and its rank	283
8.4.2	Geometric interpretation of the rank condition	284
8.4.3	Trilinear relationships among points and lines	288
8.5	Uncalibrated factorization and stratification	289
8.5.1	Equivalent multiple-view matrices	290
8.5.2	Rank-based uncalibrated factorization	291
8.5.3	Direct stratification by the absolute quadric constraint	292
8.6	Summary	294
8.7	Exercises	295
8.A	Proof for the properties of bilinear and trilinear constraints . .	305
9	Extension to General Incidence Relations	310

9.1	Incidence relations among points, lines, and planes	310
9.1.1	Incidence relations in 3-D space	310
9.1.2	Incidence relations in 2-D images	312
9.2	Rank conditions for incidence relations	313
9.2.1	Intersection of a family of lines	313
9.2.2	Restriction to a plane	316
9.3	Universal rank conditions on the multiple-view matrix	320
9.4	Summary	324
9.5	Exercises	326
9.A	Incidence relations and rank conditions	330
9.B	Beyond constraints among four views	331
9.C	Examples of geometric interpretation of the rank conditions	333
9.C.1	Case 2: $0 \leq \text{rank}(M) \leq 1$	333
9.C.2	Case 1: $1 \leq \text{rank}(M) \leq 2$	335
10	Geometry and Reconstruction from Symmetry	338
10.1	Symmetry and multiple-view geometry	338
10.1.1	Equivalent views of symmetric structures	339
10.1.2	Symmetric structure and symmetry group	341
10.1.3	Symmetric multiple-view matrix and rank condition	344
10.1.4	Homography group for a planar symmetric structure	346
10.2	Symmetry-based 3-D reconstruction	348
10.2.1	Canonical pose recovery for symmetric structure	349
10.2.2	Pose ambiguity from three types of symmetry	350
10.2.3	Structure reconstruction based on symmetry	357
10.3	Camera calibration from symmetry	364
10.3.1	Calibration from translational symmetry	365
10.3.2	Calibration from reflective symmetry	365
10.3.3	Calibration from rotational symmetry	366
10.4	Summary	367
10.5	Exercises	368
IV	Applications	373
11	Step-by-Step Building of a 3-D Model from Images	375
11.1	Feature selection	378
11.2	Feature correspondence	380
11.2.1	Feature tracking	380
11.2.2	Robust matching across wide baselines	385
11.3	Projective reconstruction	391
11.3.1	Two-view initialization	391
11.3.2	Multiple-view reconstruction	394
11.3.3	Gradient descent nonlinear refinement (“bundle adjustment”)	397

11.4	Upgrade from projective to Euclidean reconstruction	398
11.4.1	Stratification with the absolute quadric constraint	399
11.4.2	Gradient descent nonlinear refinement (“Euclidean bundle adjustment”)	402
11.5	Visualization	403
11.5.1	Epipolar rectification	404
11.5.2	Dense matching	407
11.5.3	Texture mapping	409
11.6	Additional techniques for image-based modeling	409
12	Visual Feedback	412
12.1	Structure and motion estimation as a filtering problem	414
12.1.1	Observability	415
12.1.2	Realization	418
12.1.3	Implementation issues	420
12.1.4	Complete algorithm	422
12.2	Application to virtual insertion in live video	426
12.3	Visual feedback for autonomous car driving	427
12.3.1	System setup and implementation	428
12.3.2	Vision system design	429
12.3.3	System test results	431
12.4	Visual feedback for autonomous helicopter landing	432
12.4.1	System setup and implementation	433
12.4.2	Vision system design	434
12.4.3	System performance and evaluation	436
V	Appendices	439
A	Basic Facts from Linear Algebra	441
A.1	Basic notions associated with a linear space	442
A.1.1	Linear independence and change of basis	442
A.1.2	Inner product and orthogonality	444
A.1.3	Kronecker product and stack of matrices	445
A.2	Linear transformations and matrix groups	446
A.3	Gram-Schmidt and the QR decomposition	449
A.4	Range, null space (kernel), rank and eigenvectors of a matrix	451
A.5	Symmetric matrices and skew-symmetric matrices	454
A.6	Lyapunov map and Lyapunov equation	456
A.7	The singular value decomposition (SVD)	457
A.7.1	Algebraic derivation	457
A.7.2	Geometric interpretation	459
A.7.3	Some properties of the SVD	459
B	Least-Variance Estimation and Filtering	462

B.1	Least-variance estimators of random vectors	463
B.1.1	Projections onto the range of a random vector	464
B.1.2	Solution for the linear (scalar) estimator	464
B.1.3	Affine least-variance estimator	465
B.1.4	Properties and interpretations of the least-variance estimator	466
B.2	The Kalman-Bucy filter	468
B.2.1	Linear Gaussian dynamical models	468
B.2.2	A little intuition	469
B.2.3	Observability	471
B.2.4	Derivation of the Kalman filter	472
B.3	The extended Kalman filter	476
C	Basic Facts from Nonlinear Optimization	479
C.1	Unconstrained optimization: gradient-based methods	480
C.1.1	Optimality conditions	481
C.1.2	Algorithms	482
C.2	Constrained optimization: Lagrange multiplier method	484
C.2.1	Optimality conditions	485
C.2.2	Algorithms	485
	References	487
	Glossary of Notation	509
	Index	513