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Dynamical Systems with Applications using MAPLE

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Preface

This book provides an introduction to the theory of dynamical systems with the aid of the Maple algebraic manipulation package. It is written for both senior undergraduates and first-year graduate students. The first half of the book deals with continuous systems using ordinary differential equations (Chapters 1–12) and the second half is devoted to the study of discrete dynamical systems (Chapters 13–20). (The author has gone for breadth of coverage rather than fine detail and theorems with proof are kept at a minimum.) The material is not clouded by functional analytic and group theoretical definitions, and so is intelligible to readers with a general mathematical background. Some of the topics covered are scarcely covered elsewhere. Most of the material in Chapters 9–12, 16, 17, 19, and 20 is at postgraduate level and has been influenced by the author's own research interests. It has been found that these chapters are especially useful as reference material for senior undergraduate project work. The book has a very hands-on approach and takes the reader from the basic theory right through to recently published research material.

An efficient tutorial guide to the Maple symbolic computation system has been included in Chapter 0. Students should be able to complete tutorials one and two in under two hours depending upon their past experience. The author suggests that the reader should save the relevant example programs listed throughout the book in separate files. These programs can then be edited accordingly when attempting the exercises at the end of each chapter. The Maple commands, programs and output can also be viewed in color over the Web at either http://www.birkhauser.com/cgi-win/ISBN/0-8176-4150-5

or Maple's applications site,

http://www.maplesoft.com/apps/.

Throughout the book, Maple is viewed as a tool for solving systems or producing eye-catching graphics. The author has used Maple V release 5.1 and Maple 6 in the preparation of the material. However, the Maple programs have been kept as simple as possible and should also run under later versions of the package.

The first few chapters of the book cover some theory of ordinary differential equations and applications to models in the real world are given. The theory of differential equations applied to chemical kinetics and electric circuits is introduced in some detail. Chapter 1 ends with the existence and uniqueness theorem for the solutions of certain types of differential equation. The theory behind the construction of phase plane portraits for two-dimensional systems is dealt with in Chapters 2 and 3, and applications to modeling the populations of interacting species are discussed in Chapter 4. Limit cycles, or isolated periodic solutions, are introduced in Chapter 5. Since we live in a periodic world, these are the most common type of solution found when modeling nonlinear dynamical systems. They appear extensively when modeling both the technological and natural sciences. Hamiltonian (conservative) systems and stability are discussed in Chapter 6, and Chapter 7 is concerned with how planar systems vary depending upon a parameter. Bifurcation, multistability, and bistability are discussed.

The reader is first introduced to the concept of chaos in Chapters 8 and 9, where three-dimensional systems and Poincaré maps are investigated. These higher-dimensional systems can exhibit strange attractors and chaotic dynamics. Once again the theory can be applied to chemical kinetics and electric circuits; a simplified model for the weather is also briefly discussed. Both local and global bifurcations are investigated in Chapter 10. The main results and statement of the famous second part of David Hilbert's sixteenth problem are listed in Chapter 11. In order to understand these results, Poincaré compactification is introduced. The study of continuous systems ends with one of the authors specialities—limit cycles of Liénard systems. There is some detail on Liénard systems in particular in the first half of the book, but they do have a ubiquity for systems in the plane.

Chapters 13–20 deal with discrete dynamical systems. Chapter 13 starts with a general introduction to recurrence relations and iteration. Applications to population modeling and harvesting and culling policies is then investigated. Chaos in discrete systems is investigated and bifurcation diagrams are plotted in Chapter 14. The concept of universality is discussed for the first time. Complex iterative maps are introduced in Chapter 15. Julia sets and the now famous Mandelbrot set are plotted. As a simple introduction to optics, electromagnetic waves and Maxwell's equations are studied at the beginning of Chapter 16. A brief history of nonlinear bistable optical resonators is discussed and the simple fiber ring resonator is analyzed in particular. Chapters 16 and 17 are devoted to the study of these optical resonators and topics such as bifurcation, bistability, chaos, chaotic attractors, instabilities, linear stability analysis, multistability, and nonlinearity, which have already been dealt with in earlier chapters, are reviewed. Some simple fractals may be constructed using pencil and paper in Chapter 18, and the idea of fractal dimension is introduced. Fractals may be thought of as identical motifs repeated on ever reduced scales. Unfortunately, most of the fractals appearing in nature are not homogeneous but are more heterogeneous, hence the need for the multifractal theory given in Chapter 19. The final chapter is devoted to the new and exciting theory behind chaos control. For most systems, the maxim used by engineers in the past has been "stability good, chaos bad," but more and more nowadays this is being replaced with "stability good, chaos better." There are exciting and new applications to cardiology, laser technology, and space research, for example.

This book is informed by the research interests of the author which are currently nonlinear ordinary differential equations, nonlinear optics and multifractals. Some references include recently published research articles.

The prerequisites for studying dynamical systems using this book are undergraduate courses in linear algebra, real and complex analysis, calculus and ordinary differential equations; a knowledge of a computer language such as Fortran or Pascal would be beneficial but not essential.

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