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Joseph Liouville 1809-1882: Master of Pure and Applied Mathematics

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Jesper Lützen

Joseph Liouville 1809–1882:
Master of Pure and Applied Mathematics

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Jesper Lützen
Department of Mathematics
University of Copenhagen
DK-2100 Copenhagen
Denmark

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To my parents

J. Liouville

Preface

Joseph Liouville was the most important French mathematician in the generation between Galois and Hermite. This is reflected in the fact that even today all mathematicians know at least one of the more than six theorems named after him and regularly study *Liouville's Journal*, as the *Journal de Mathématiques pures et appliquées* is usually nicknamed after its creator. However, few mathematicians are aware of the astonishing variety of Liouville's contributions to almost all areas of pure and applied mathematics. The reason is that these contributions have not been studied in their historical context. In the *Dictionary of Scientific Biography* 1973, Taton [1973] gave a rather sad but also true picture of the Liouville studies carried out up to that date:

> The few articles devoted to Liouville contain little biographical data. Thus the principal stages of his life must be reconstructed on the basis of original documentation. There is no exhaustive list of Liouville's works, which are dispersed in some 400 publications.... His work as a whole has been treated in only two original studies of limited scope those of G. Chrystal and G. Loria.

Since this was written, the situation has improved somewhat through the publications of Peiffer, Edwards, Neuenschwander, and myself. Moreover, C. Houzel and I have planned on publishing Liouville's collected works. However, considering Liouville's central position in French mathematical science in the middle of the 19th century, there is, I think, still a need for a more comprehensive study of his life and work. I hope that I have been able to remedy this neglect in part by bringing together the published evidence and the extensive archival material, in particular Liouville's *Nachlass* of 340 notebooks, consisting of more than 40,000 pages.

When I started this work in 1980, I had the ambition of writing *the* definitive biography of Liouville. Now, however, I know that the result is only a modest first approximation, which I hope will be extended by other historians and mathematicians. In particular, I have not gone as deeply into Liouville's private life, family matters, etc., as I had planned to do. The reason is that after the initial archival studies I discovered that Neuenschwander was doing the same job. This means, for example, that I have not used the Bordeaux archive that Neuenschwander had discovered

[Neuenschwander 1984a, note 5]. Neither have I studied the newspapers of the time.

The more limited aim of this book is to tell the story of Liouville's scientific career: his education and his work as a teacher, journal editor, politician, and academician, and not least to analyze his mathematical works and place them in a historical perspective.

The book consists of two parts. The first (Chapters I–VI) is a chronological account of Liouville's career. We follow him from his student days at the *École Polytechnique* and the *École des Ponts et Chaussées* to his glorious days as a celebrated academician and professor at the most prestigious schools in Paris. We follow his less successful and brief career as a politician during the Second Republic, his renewed scientific creativity during the 1850s, and finally his last years full of illness, pain, and misery.

In this connection, I have tried to give a picture of the rather complex institutional setting in Paris. This part also contains descriptions of Liouville's teachers, colleagues, and students, both French and foreign, and of his two scientific archenemies, Libri and Le Verrier (whose name, by the way, I shall spell the way Liouville usually did, i.e., with a capital V). Liouville's private life is of course also taken into account, in particular when it tells us something about his scientific career.

Liouville's mathematical and physical works also have a central place in the first part. Indeed, one can consider the first part as an explanation of how Liouville's various works fit into a global picture of his life. Yet, the description of the mathematics is not technical in the first part and is mostly without formulas.

In the second part, one can find a more thorough analysis of various aspects of Liouville's work. This part is divided into 10 chapters concentrating on different mathematical and physical fields, of which Chapters X and XI are slightly revised versions of two previously published papers [Lützen 1984a,b]. Thus, the chronology is broken. It has been my aim that these chapters can be read independently of each other and independently of Part I of the book. This means that mathematicians can concentrate on those particular parts of Liouville's production they like the best, but it also means that I have been forced to repeat certain things.

The mathematical and historical analysis of Liouville's work relies heavily on his notebooks; for not only do these reveal the genesis of many of his published results, they also contain many ingenious ideas, methods, and results that he did not publish. It is in fact remarkable that many of Liouville's most far-reaching ideas were never published.

The mathematical analysis of Liouville's notes has not only been the most challenging but also the most interesting and rewarding work involved in the research concerned with this biography. The bulk of the material is overwhelming, and the notes are usually written pell-mell among each other and often in a very sketchy style, which even made them hard for Liouville himself to understand. In a letter to Bertrand concerning a passage by Laplace on tautochrones, Liouville wrote:

> Now I have found the note that I wrote on this subject at least twelve years ago; it is terrifying (you are too young to know that) to see that as time passes one forgets what one has known the best. My note which is part of a sort of commentary on the first two volumes of the *mécanique céleste* that I composed for my own use around 1834 and 1835 and which are full of abbreviations, which I now have some difficulty in understanding...
>
> [Neuenschwander 1984a, III, 2]

Similarly, in a letter to Dirichlet [Tannery 1910, p. 40], Liouville admitted that his notebooks "are in complete disorder." These quotes have often comforted me when I was unable to understand the content, or even the subject, of a particular note. I have looked through the notebooks a few times in order to find interesting entries. Many of the notes, in particular in the last 200 notebooks, are clearly without much mathematical interest, but there may very well be hidden treasures that I have overlooked. Indeed, I suspect that one can find valuable information about Liouville's ideas on number theory. Since I have no knowledge of number theory, I have omitted an analysis of this perplexing chapter of Liouville's production, but otherwise I have tried to discuss all of Liouville's major contributions to mathematics and physics.

The published and, in particular, the unpublished materials have allowed me to make several observations concerning Liouville's work that I do not think have been made before. Here, I shall list a few of the more remarkable conclusions:

1. Liouville began his career as a mathematical physicist writing on electrodynamics and the theory of heat.

2. His works on electrodynamics probably motivated his research on the fractional calculus.

3. His works on the theory of heat led to his first work on Sturm-Liouville theory.

4. His generalization of Sturm-Liouville theory to a higher order partly failed because he tried to prove the completeness of a system of eigenfunctions that is not complete.

5. Liouville's approach to integration in finite terms developed from being analytical to being algebraic.

6. Liouville knew a theorem on integration in implicit finite terms, which was proved by Risch as late as 1976, and he even had a formally correct proof of it.

7. In 1842, Liouville developed a theory of the stability of rotating masses of fluids that shares many methods and results in common with the published theories by Poincaré and Liapounoff from the mid-1880s.

8. Liouville's proof of the existence of transcendental numbers was inspired by a correspondence between Goldbach and Daniel Bernoulli and was based on Lagrange's formulas for continued fractions.

9. One can reconstruct how Liouville found Liouville's theorem on doubly periodic functions from theorems on trigonometric series.

10. In his unpublished, notes Liouville succeeded in filling the most conspicuous holes in Galois's great memoir.

11. Starting from potential theory, Liouville, in the 1840s, created a rather general theory of integral operators, which anticipated Hilbert's ideas. In particular, it was based on the Rayleigh-Ritz method of finding eigenvalues.

12. This theory and Liouville's work on Sturm-Liouville theory show that he was the first great expert in spectral theory.

13. Liouville wrote an unpublished memoir on a generalization of the Poisson and Lagrange brackets. It led him to a theory of transformations of the equations of mechanics.

14. Liouville may have been the mathematician before Riemann who best understood the value of an intrinsic formulation of differential geometry.

15. This insight allowed him to use geometric ideas in an interesting transformation of the principle of least action. In two variables, he saw that trajectories of a particle acted on by forces can be thought of as geodesics on another surface. His general transformation theorem stopped short of generalizing this to the observation that trajectories are nothing but geodesics in a suitable Riemannian metric.

These somewhat crudely formulated conclusions are amplified and documented in the chapters of Part II.

Generally, Liouville's production is characterized by a slow movement from applied to pure mathematics; here, Liouville followed a general tendency in mathematics of the time. Most of his life he continued to be inspired by physical problems, although Laboulay, in his speech at Liouville's funeral, argued that this was a self-imposed restraint from the most abstract spheres where he felt at home as well:

> His mind operated in these heights where few savants could follow him. He said jokingly that there had been a problem that could only be posed or understood by three initiates in the entire world: a Russian savant, an American lady and a third mathematician whose name he did not mention; but this was not the subject of the sciences, and he added that there were such problems that could only be understood by two people. It was Liouville himself who because of modesty renounced rising to this last summit of abstraction.
>
> [Laboulay 1882]

As a person, Liouville seems to have been helpful and kind, but also firm in his beliefs. Although a biographer should perhaps keep clear of

such feelings, I cannot help liking him and feeling sorry for him when life became unendurable to him. I hope, however, that this has not made me too biased.

I have tried to write the first part in a narrative style, whereas the second part is more discursive. In an attempt to make the text easy to read, I have tried to avoid footnotes as much as possible. For the same reason, I have translated all the French quotes (except a few quotes in the appendices). This causes a problem of documentation, in particular concerning the unpublished material that the reader cannot easily find in the original. I contemplated appending all the original French quotes, but that would have made this already long book even longer. Only the excerpts from the *Procès Verbaux du Bureau des Longitudes* are quoted in the original as well, because the president of the Bureau has asked me to do that.

References to places in this book are included in parentheses, e.g., (Chapter XV, §8); the chapters carry Roman numerals, and the number of the section is preceded by a §. Only the chapters in the second part of the book are divided into numbered sections. Therefore, I refer to places in the first part by page number, e.g. (Chapter III, p. 10). Formulas are consecutively numbered in each chapter. A reference to a formula appears as (Chapter XV (6)) or just (XV (6)). When I refer to a formula or a section in the same chapter, I omit the number of the chapter, e.g., (6) or §8.

The formulas in some of the quotations are renumbered to fit into the consecutive numbering of the other formulas in the chapter. Such numbers are put in square brackets, just as all other additions to the original text.

Otherwise, square brackets are used for references to the works mentioned in the bibliography. These references are given by the author's name and year of publication. In some instances, the year of composition or of presentation to an academy is used. If the bibliography contains several publications by one author from the same year, these are labeled a, b, c,..., e.g., [Liouville 1836b]. In cases where it is clear which author is cited, only the year is given in square brackets. Three abbreviations have been used throughout: C. R. for the *Comptes Rendus des Séances de l'Académie des Sciences de Paris*, L. J. for the *Journal de Mathématiques pures et appliquées*, or Liouville's Journal as it is often called, and Ms for the manuscripts in Liouville's *Nachlass* at the *Bibliothèque de l'Institut de France*. I have paginated many of the first notebooks but not the last ones. Therefore, only some references to the Ms carry page numbers.

References to archival material are usually given by some easily recognizable abbreviation, e.g., P. V. Bur. Long. for the *Procès Verbaux du Bureau des Longitudes*. The manuscripts found at the *Archive de l'Académie des Sciences*, however, have been given a special code. This code refers to the list of manuscripts at the beginning of the bibliography. The bibliography continues with a list of Liouville's published works and ends with the publications by other authors.

Years of birth and death of the persons mentioned in the book can be found in the index, at least as far as I have been able to find them.

Acknowledgments: During the eight years of writing this book, I have held appointments at two departments: first at the Mathematical Department at the University of Odense, and then at the Mathematical Department at the University of Copenhagen. I am grateful to my colleagues and the staff at these institutions for their kind encouragement and help. In particular, I wish to thank Uffe Haagerup and Christian Berg for having cleared up some difficult mathematical points. All along I have been supported from my former teachers and colleagues at the Institute for the History of Exact Sciences at the University of Aarhus and in particular by Kirsti Andersen, whose usual kindness, constructive criticism, and encouragement have been of great help.

During my stays in Paris, which were partly funded by the Danish Royal Society and the French State, I was fortunate to get a room in the Fondation Danoise. My research there was also greatly facilitated by the helpfulness of the staff at the libraries and archives I visited, such as the *Archive de l'Académie des Sciences*, the *Bureau des Longitudes*, whose President Mr. Kergrohen kindly allowed me to read through their *Procès Verbaux*, the *Archive de l'École Polytechnique*, the *Archive de la Collège de France*, and not the least, the *Bibliotèque de l'Institut de France*, where I spent most of my time in Paris studying Liouville's notebooks. I am grateful to these institutions and individuals for their kind help.

I also wish to thank Bruno Belhoste for having helped me find my way in the Parisian archival jungle and Professor F. Jongmans and Jan-Erik Roos, who provided me with valuable material concerning Catalan, Bjerknes, and Holmgren; I am also grateful to many other colleagues from the generalized Oberwolfach circle, in particular, to Ivor Grattan-Guinness and Harold M. Edwards for their helpful suggestions concerning various points of this book.

Finally, my appreciation goes to Jeremy Gray who has made my broken English sound less broken, to Dita Andersen, who enthusiastically has struggled transforming my scrawl into a TEX file, and to Springer-Verlag for their generous editorial care. It was their Mathematics Editor, W. Kaufmann-Buhler, who originally suggested, at a very early stage of my work, that I should contact him as soon as my plans had developed far enough. I am very sorry that he did not live to express his opinion on my book, for after having read his fine biography of Gauss, I am sure he would have suggested valuable changes.

Copenhagen
June 1988

Jesper Lützen

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