Contents

Preface ix

Chapter 1 **Finite and Infinite Words** 1

1.0 Introduction 1
1.1 Semigroups 1
1.2 Words 3
1.3 Automata 12
1.4 Generating series 22
1.5 Symbolic dynamical systems 26
1.6 Unavoidable sets 33
Problems 40
Notes 44

Chapter 2 **Sturmian Words** 45

2.0 Introduction 45
2.1 Equivalent definitions 46
2.2 Standard words 63
2.3 Sturmian morphisms 83
Problems 101
Notes 108

Chapter 3 **Unavoidable Patterns** 111

3.0 Introduction 111
3.1 Definitions and basic properties 112
3.2 Deciding avoidability: the Zimin algorithm 116
3.3 Avoidability on a fixed alphabet 123
Problems 131
Notes 133

Chapter 4 Sesquipowers 135

4.0 Introduction 135
4.1 Bi-ideal sequences 136
4.2 Canonical factorizations 137
4.3 Sesquipowers and recurrence 141
4.4 Extensions of a theorem of Shirshov 144
4.5 Finiteness conditions for semigroups 149
Problems 159
Notes 162

Chapter 5 The Plactic Monoid 164

5.0 Introduction 164
5.1 Schensted's algorithm 165
5.2 Greene's invariants and the plactic monoid 167
5.3 The Robinson–Schensted–Knuth correspondence . . . 170
5.4 Schur functions and the Littlewood–Richardson rule . 176
5.5 Coplactic operations 182
5.6 Cyclage and canonical embeddings 186
Problems 192
Notes 195

Chapter 6 Codes 197

6.0 Introduction 197
6.1 X-factorizations 198
6.2 Defect 204
6.3 More defect 209
6.4 A theorem of Schützenberger 221
Problems 224
Notes 227

Chapter 7 Numeration Systems 230

7.0 Introduction 230
7.1 Standard representation of numbers 231
7.2 Beta-expansions 236
7.3 U-representations 248
7.4 Representation of complex numbers 257
Problems 263
Notes 267

Chapter 8 Periodicity 269

8.0 Introduction 269
8.1 Periods in a finite word 270
8.2 Local versus global periodicity 283
8.3 Infinite words 294
Problems 306
Notes 310

Chapter 9 Centralizers of Noncommutative Series and Polynomials 312

9.0 Introduction 312
9.1 Cohn's centralizer theorem 313
9.2 Euclidean division and principal right ideals 315
9.3 Integral closure of the centralizer 316
9.4 Homomorphisms into $k[t]$ 319
9.5 Bergman's centralizer theorem 320
9.6 Free subalgebras and the defect theorem 322
9.7 Appendix: some commutative algebra 324
Notes 328

Chapter 10 Transformations on Words and q-Calculus 330

10.0 Introduction 330
10.1 The q-binomial coefficients 332
10.2 The MacMahon Verfahren 334
10.3 The insertion technique 337
10.4 The (t, q)-factorial generating functions 339
10.5 Words and biwords 341
10.6 Commutations 343
10.7 The two commutations 345
10.8 The main algorithm 347
10.9 The inverse of the algorithm 349
10.10 Statistics on circuits 351
10.11 Statistics on words and equidistribution properties . . . 353
Problems 355
Notes 363

Chapter 11 Statistics on Permutations and Words 365

11.0 Introduction 365
11.1 Preliminaries 366
11.2 Words with a given shape 367
11.3 Backsteps of permutations with a given shape 369

11.4 Inversions of permutations with a given shape 372
11.5 Lyndon factorization and cycles of permutations 375
11.6 Major index of permutations with a given cyclic type . 378
Problems 381
Notes 385

Chapter 12 Makanin's Algorithm 387

12.0 Introduction 387
12.1 Words and word equations 389
12.2 The exponent of periodicity 400
12.3 Boundary equations 406
12.4 Proof of Theorem 12.3.10 432
Problems 435
Notes 438

Chapter 13 Independent Systems of Equations 443

13.0 Introduction 443
13.1 Sets and equations 444
13.2 The compactness property 447
13.3 Independence of finite systems of equations 453
13.4 Semigroups without the compactness property 458
13.5 Semigroups with the compactness property 460
Problems 468
Notes 470

References 473

Index of Notation 497

General Index 499