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(continued after index)

J. David Logan

Applied Partial Differential Equations

With 35 Illustrations

Springer

J. David Logan
Department of Mathematics and Statistics
University of Nebraska at Lincoln
Lincoln, NE 68588-0323
USA

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Preface

This textbook is for the standard, one-semester, junior–senior course that often goes by the title "Elementary Partial Differential Equations" or "Boundary Value Problems." The audience usually consists of students in mathematics, engineering, and the physical sciences. The topics include derivations of some of the standard equations of mathematical physics (including the heat equation, the wave equation, and the Laplace's equation) and methods for solving those equations on bounded and unbounded domains. Methods include eigenfunction expansions or separation of variables, and methods based on Fourier and Laplace transforms. Prerequisites include calculus and a post-calculus differential equations course.

There are several excellent texts for this course, so one can legitimately ask why one would wish to write another. A survey of the content of the existing titles shows that their scope is broad and the analysis detailed; and they often exceed five hundred pages in length. These books generally have enough material for two, three, or even four semesters. Yet, many undergraduate courses are one-semester courses. The author has often felt that students become a little uncomfortable when an instructor jumps around in a long volume searching for the right topics, or only partially covers some topics; but they are secure in completely mastering a short, well-defined introduction. This text was written to provide a brief, one-semester introduction to partial differential equations. It is limited in both scope and depth compared with existing books, yet it covers the main topics usually studied in the standard course and also provides an introduction to using computer algebra packages to solve and understand partial differential equations. The frontiers of mathematics and science are receding rapidly, and a one-semester course must try to advance the students to a level where they can reach these boundaries more quickly than in the past. Not every traditional topic can be covered, and not every topic can be examined in great detail. An example is the method of separation of variables, which plays a dominant role in most texts. It is this author's view that a few well-chosen illustrations of the method of separation of variables should suffice.

The level of exposition in this text is slightly higher than one usually encounters in the post-calculus differential equations course. The philosophy here is that a student should progress in his or her ability to read mathematics. Elementary calculus texts, and even ordinary differential equations texts, contain lots of examples and detailed calculations, but advanced mathematics and science books leave a lot to the reader. This text leaves some of the details that are easy to supply to the reader. The student is encouraged as part of the learning process to fill in these missing details (see "To the Student"). The writing has more of an engineering and science style to it than a traditional, mathematical, theorem–proof format. Consequently, the arguments given are "derivations" in lieu of carefully constructed proofs.

Students who come out of reform calculus courses often have different skills from those of students who have had a traditional calculus course. For example, they rely more on calculators and computers in solving some of their problems. This text is a little less traditional than other elementary texts on partial differential equations. It encourages the student to use software packages, where applicable, in solving problems. Herein we use Maple, version V4, for the illustrations, but the text is in no way dependent on Maple; Mathematica, or any other computer algebra program, can be used. The exercises encourage students to think about the concepts and derivations rather than just grind out lots of routine solutions. The student who reads this book carefully and who solves most of the exercises will have a sound enough knowledge base to continue with a second-year partial differential equations course where careful proofs are constructed or with upper-division courses in science and engineering where detailed applications of partial differential equations are introduced. Both the exposition and exercises will build analytical skills that some students did not develop in the reform calculus courses.

In Chapter 1 we introduce some basic partial differential equations of applied mathematics. Many of the basic equations come from a conservation law, or balance law, and describe physical processes like convection, diffusion, and reaction. There are a variety of applications to show the central role that partial differential equations play in science, engineering, and mathematics. Applications include contaminant transport in aquifers, convection and diffusion processes in biology and chemical engineering, quantum mechanics, mechanical vibrations, heat flow, electromagnetic phenomena, and acoustics. The chapter ends with a discussion of classification of partial differential equations; the emphasis is on the physical meaning of the equations (diffusion-like, wave-like, etc.) as well as on the analytical structure of the equations. The goal of the chapter is to give students a sense of the origins of partial differential equations and how their solutions differ. At the same time the exercises force the students to revisit the chain rule, the divergence theorem, and other concepts from multivariable calculus.

Chapter 2 examines equations on unbounded domains, both infinite and semi-infinite. The author's view is that these problems are simpler than their counterparts on bounded domains with boundaries present. Easy formulae are obtained for solutions to the initial value problems for the heat and wave equations, and well-posedness is discussed. Problems with sources are handled with Duhamel's principle. Most students have studied Laplace transforms in an elementary ordinary differential equations course, so it is a natural transition to study transform methods for partial differential equations; the Fourier transform is a straightforward extension from the Laplace transform. We also include material on using computer algebra packages to find general solutions to equations and solutions based on transform methods.

One of the fundamental ideas in applied mathematics is orthogonality. In Chapter 3, rather than adopt a strict focus on Fourier series, a general strategy is taken. Calculus courses have always included Taylor series, and many calculus courses, especially reform courses, now include some material on Fourier series. Therefore, students are ready to be introduced to general expansions of functions in series, especially orthogonal series. These expansions are motivated by the separation of variables method, and classical Fourier series are studied as a special case. The chapter ends with a study of Sturm–Liouville problems.

Chapter 4 contains traditional material on the separation of variables method for solving partial differential equations on bounded domains. Here we solve various equations with various boundary conditions in rectangular, cylindrical, and spherical geometry. Students are urged to use software packages to perform some of the calculations. There is a section on inverse problems and a long section on the finite difference method.

I want to thank my wife, Tess, and my son, David, for their encouragement and support. I also thank my many students for enduring preliminary versions of this text and especially for their comments on the clarity of the exposition and suitability of some of the exercises.

Suggestions for Using the Text: The author has taught this material on numerous occasions and uses approximately the following schedule: Chapter 1 (12 classes), Chapter 2 (11 classes), Chapter 3 (7 classes), Chapter 4 (14 classes). Under this schedule, the sections marked with an asterisk (*) in the Table of Contents are sometimes not covered in lectures. Often those sections are assigned as extra reading material to graduate students taking the course.

Lincoln, Nebraska J. David Logan

To the Student

Partial differential equations (PDEs) is a subject about differential equations for unknown functions of several variables; the derivatives involved are partial derivatives. As such, it is a subject that is intimately connected with multivariable or third-semester calculus. To be successful you should have, first, a good command of the concepts in the calculus of several variables. So keep a calculus text nearby and review concepts when they are needed. The same comments apply to elementary ordinary differential equations (ODEs). There is an appendix at the end of the book that reviews some of the basic solution techniques for ODEs.

Second, a mathematics book should be read with a pencil and paper at hand. Elementary books fill in most of the steps in the exposition, but advanced books leave many details to the reader. This book has enough detail so that you can follow the discussion, but pencil and paper work is required in some portions. Verifying all the statements in a text is a worthwhile endeavor and will help you learn the material. Many students find that studying PDEs provides an opportunity to reinforce many calculus concepts and calculations.

Finally, the exercises are an important part of this text (perhaps the most important part!), and you should try to solve most or all of them. Some will require routine analytical or computer calculations, but others will require careful thought. We learn mathematics by doing mathematics, even when we are stymied by a problem. The effort put into a failed attempt will help you sort out the concepts and reinforce the learning process. View the exercises as a challenge and resist the temptation to give up.

Lincoln, Nebraska — J. David Logan

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