

Brooks Roberts      Ralf Schmidt

# Local Newforms for $\mathrm{GSp}(4)$

 Springer

---

## Contents

<b>1</b>	<b>A Summary</b> .....	1
<b>2</b>	<b>Representation Theory</b> .....	27
2.1	Definitions .....	27
2.2	Parabolically Induced Representations .....	34
2.3	Dual Groups .....	40
2.4	The Local Langlands Correspondence .....	46
2.5	$P_3$ -Theory .....	62
2.6	Zeta Integrals .....	74
<b>3</b>	<b>Paramodular Vectors</b> .....	85
3.1	Linear Independence .....	85
3.2	The Level Raising Operators $\theta$ , $\theta'$ and $\eta$ .....	90
3.3	Level Lowering Operators .....	107
3.4	Paramodular Vectors and $P_3$ -Theory .....	117
<b>4</b>	<b>Zeta Integrals</b> .....	123
4.1	Paramodular Vectors and Zeta Integrals .....	123
4.2	Poles and $P_3$ -Theory .....	128
4.3	The $\eta$ Principle .....	135
4.4	The Existence Theorem for Generic Representations .....	148
<b>5</b>	<b>Non-supercuspidal Representations</b> .....	151
5.1	Double Coset Decompositions .....	151
5.2	Induction from the Siegel Parabolic Subgroup .....	156
5.3	Representations of Type IIIb and IVc .....	161
5.4	Induction from the Klingen Parabolic Subgroup .....	165
5.5	Saito–Kurokawa Representations .....	168
5.6	Summary .....	182
5.7	Atkin–Lehner Eigenvalues .....	183

<b>6</b>	<b>Hecke Operators</b> . . . . .	187
6.1	Two Hecke Operators . . . . .	188
6.2	The Commutation Relation . . . . .	195
6.3	Hecke Operators and Level Raising . . . . .	205
6.4	Computation of Hecke Eigenvalues . . . . .	213
6.5	Some Consequences of Unitarity . . . . .	236
<b>7</b>	<b>Proofs of the Main Theorems</b> . . . . .	239
7.1	Zeta Integrals: The Unramified Case . . . . .	239
7.2	Zeta Integrals: The Level $\mathfrak{p}$ Case . . . . .	243
7.3	The Operator $R$ . . . . .	248
7.4	Zeta Integrals: The Higher Level Case . . . . .	252
7.5	Main Results . . . . .	262
<b>A</b>	<b>Tables for Representations of <math>\mathbf{GSp}(4)</math></b> . . . . .	269
A.1	Non-supercuspidal Representations . . . . .	269
A.2	Unitary Representations . . . . .	271
A.3	Jacquet Modules . . . . .	273
A.4	The $P_3$ -Filtration . . . . .	277
A.5	$L$ -Parameters . . . . .	280
A.6	$L$ - and $\varepsilon$ -factors (degree 4) . . . . .	282
A.7	$L$ - and $\varepsilon$ -factors (degree 5) . . . . .	286
A.8	Paramodular Dimensions and Atkin–Lehner Eigenvalues . . . . .	290
A.9	Hecke Eigenvalues . . . . .	294
A.10	Parahori-invariant Vectors . . . . .	297
	<b>References</b> . . . . .	301
	<b>Index</b> . . . . .	305