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I-Shih Liu

Continuum Mechanics

With 28 Figures and Numerous Exercises



Springer

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To my teacher
Ingo Müller

Preface

In this book the basic principles of continuum mechanics and thermodynamics are treated in the tradition of the rational framework established in the 1960s, typically in the fundamental memoir “The Non-Linear Field Theories of Mechanics” by Truesdell and Noll. The theoretical aspect of constitutive theories for materials in general has been carefully developed in mathematical clarity – from general kinematics, balance equations, material objectivity, and isotropic representations to the framework of rational thermodynamics based on the entropy principle. However, I make no claim that the subjects are covered completely, nor does this book cover solutions and examples that can usually be found in textbooks of fluid mechanics and linear elasticity. However, some of the interesting examples of finite deformations in elastic materials, such as biaxial stretching of an elastic membrane and inflation of a rubber balloon, are discussed.

In the last two chapters of the book, some recent developments in thermodynamic theories are considered. Specifically, they emphasize the use of Lagrange multipliers, which enables the exploitation of the entropy principle in a systematic manner for constitutive equations, and introduce some basic notions of extended thermodynamics. Although extended thermodynamics is closely related to the kinetic theory of ideal gases, very limited knowledge of kinetic theory is needed.

Earlier versions of this book have been used over the years, in the Institute of Mathematics at the Federal University of Rio de Janeiro as well as in the Institute of Applied Mechanics at the National Taiwan University, in an introductory course on continuum mechanics at the graduate level, and at the advanced undergraduate level with a simplified version. The readers are not required to have a good knowledge of either solid mechanics or fluid mechanics, but, of course, some prior acquaintance with them would be helpful.

An appendix is written at the end to provide a review of basic notions in linear algebra and tensor analysis as mathematical preliminaries for the subjects, and occasionally cross-references to it (e.g. (A.32)) are used in the text. The reader who already has a reasonable mathematical knowledge may refer to it for reference and notations. However, in introductory courses I have often put the appendix before the first chapter because most of the students may not be familiar with the notations and some basic notions. No

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effort has been made to compile an extensive bibliography on related works in continuum mechanics. Only those cited in the book are listed.

Examples and exercises are given to supplement the understanding of the material and sometimes to provide further insights into the subjects. Usually my students are asked to do most of the exercises to accompany the progress of learning. Their feedback on the errors and the difficulties has resulted in considerable improvement of the manuscript. Their participation is greatly appreciated.

The endeavor of writing this book depended on many ideas and work in the scientific literature. To many of the relevant researchers, acquaintances or not, are due my grateful acknowledgements for their contributions. My special acknowledgement is due to Prof. Müller for his friendship and inspirations on many of my scientific trajectories. Finally, I would like to thank my family, especially my wife Lu Ping, for their understanding and patience during many long hours of preparing the manuscript over the years.

Rio de Janeiro,
March 2002

I-Shih Liu

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