

Series in Measurement Science and Technology

Evaluating the Measurement Uncertainty

Fundamentals and Practical Guidance

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With an introductory chapter by T J Quinn, Director of the BIPM

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