

Klaus Metsch

# Linear Spaces with Few Lines

**Springer-Verlag**

Berlin Heidelberg New York  
London Paris Tokyo  
Hong Kong Barcelona  
Budapest

Author

Klaus Metsch  
Mathematisches Institut  
Justus-Liebig-Universität  
Arndtstraße 2, W-6300 Gießen, FRG

Mathematics Subject Classification (1991): 51E20

ISBN 3-540-54720-7 Springer-Verlag Berlin Heidelberg New York  
ISBN 0-387-54720-7 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1991  
Printed in Germany

Typesetting: Camera ready by author  
Printing and binding: Druckhaus Beltz, Hemsbach/Bergstr.  
46/3140-543210 - Printed on acid-free paper

## C O N T E N T S

1.	Definitions and basic properties of linear spaces	1
2.	Lower bounds for the number of lines	9
3.	Basic properties and results on $(n+1,1)$ -designs	15
4.	Points of degree $n$	21
5.	Linear spaces with few lines	31
6.	Embedding $(n+1,1)$ -designs in projective planes	43
7.	An optimal bound for embedding linear spaces into projective planes	61
8.	The Theorem of Totten	74
9.	Linear spaces with $n^2+n+1$ points	86
10.	A hypothetical structure	94
11.	Linear spaces with $n^2+n+2$ lines	106
12.	Points of degree $n$ and another characterization of the linear spaces $L(n,d)$	118
13.	The non-Existence of certain $(7,1)$ -designs and determination of $A(5)$ and $A(6)$	131
14.	A result on graph theory with an application to linear spaces	141
15.	Linear spaces in which every long line meets only few other lines	150
16.	$s$ -fold inflated projective planes	161
17.	The Dowling-Wilson Conjecture	181
18.	Uniqueness of embeddings	188
	References	192
	Index	196